Question 1: Exotic light quarks

We consider a model with the gauge symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y$ spontaneously broken by a single Higgs doublet into $SU(3)_C \times U(1)_{EM}$. However, quark representations differ from the standard model

$$Q_L(3,2)_{1/6} \qquad S_L(3,1)_{-1/3} \qquad Q_R(3,2)_{1/6} \qquad S_R(3,1)_{-1/3} \tag{1}$$

1. Write down (a) the gauge interactions of the quarks with the charged W bosons; (b) the Yukawa interactions and the bare mass terms of the various quarks (before SSB); (c) the mass terms after SSB.
   Answer: The charged interaction is

$$-\mathcal{L}_W = \frac{g}{\sqrt{2}} \left[\bar{u}_L \gamma^\mu d_L + \bar{u}_R \gamma^\mu d_R\right] W^+_\mu + h.c. \tag{2}$$

   The Yukawa interactions are

$$-\mathcal{L}_Y = G_L \begin{pmatrix} \bar{u}_L & \bar{d}_L \end{pmatrix} \phi \; s_R + G_R \bar{s}_L \phi \begin{pmatrix} u_R \\ d_R \end{pmatrix} + h.c. \tag{3}$$

   The bare mass terms are

$$-\mathcal{L}^{bare}_m = m_{ss} \bar{s}_L s_R + m_{QQ} \begin{pmatrix} \bar{u}_L & \bar{d}_L \end{pmatrix} \begin{pmatrix} u_R \\ d_R \end{pmatrix} + h.c. \tag{4}$$

   After SSB the mass terms are

$$-\mathcal{L}_m = \begin{pmatrix} \bar{d}_L & \bar{s}_L \end{pmatrix} \begin{pmatrix} G_L v/\sqrt{2} & m_{QQ} \\ m_{ss} & G_R v/\sqrt{2} \end{pmatrix} \begin{pmatrix} d_R \\ s_R \end{pmatrix} + m_{QQ} \bar{u}_L u_R + h.c. \tag{5}$$

2. Write down the charged gauge interactions in the mass basis. Is there charged current for left handed fermions? Is there for right handed fermions?
   Answer:

$$\frac{-g}{\sqrt{2}} \left[\bar{u}_L \gamma^\mu \begin{pmatrix} \cos\theta_L & \sin\theta_L \end{pmatrix} \begin{pmatrix} d_L \\ s_L \end{pmatrix} + \bar{u}_R \gamma^\mu \begin{pmatrix} \cos\theta_R & \sin\theta_R \end{pmatrix} \begin{pmatrix} d_R \\ s_R \end{pmatrix}\right] W^+_\mu + h.c. \tag{6}$$

   where θ_L (θ_R) is the angle that parameterize the rotation of the left (right) handed fields from the interaction to the mass basis. Note that we have charged current interactions for both RH and LH quarks.

3. How many physical flavor parameters are in this model? Separate them into masses, mixing angles and phases. Is there CP violation in this model?

Answer: There are 4 Yukawa and bare mass terms, thus there are 8 parameters. The flavor symmetry of the kinetic term is $U(1)^4$. The flavor symmetry of the model is $U(1)$ (baryon number). The number of broken generators is $4-1=3$. The number of physical parameters is $8-3=5$. They are the three masses and the two mixing angles θ_L and θ_R. There are no phases, and thus no CP violation.

4. Write down the gauge interactions of the quarks with the Z boson in both the interaction basis and the mass basis. (You do not have to rewrite terms that do not change when you rotate to the mass basis. Write only the terms that are modified by the rotation to the mass basis.) Are there generally tree level Z exchange FCNC's?
Answer: Recalling that the coupling of the Z is $T_3 - \sin^2\theta_W q$ we have

$$\begin{aligned} -\mathcal{L}_Z = \frac{g}{\cos\theta_W} \quad & \left[\bar{u}_L\gamma^\mu\left(\frac{1}{2}-\frac{2}{3}\sin^2\theta_W\right)u_L + \bar{u}_R\gamma^\mu\left(\frac{1}{2}-\frac{2}{3}\sin^2\theta_W\right)u_R\right. \\ & \bar{s}_L\gamma^\mu\left(-\frac{1}{2}+\frac{1}{3}\sin^2\theta_W\right)s_L + \bar{d}_R\gamma^\mu\left(-\frac{1}{2}+\frac{1}{3}\sin^2\theta_W\right)d_R \\ & \left.\bar{s}_L\gamma^\mu\left(\frac{1}{3}\sin^2\theta_W\right)s_L + \bar{s}_R\gamma^\mu\left(\frac{1}{3}\sin^2\theta_W\right)s_R\right] Z_\mu \end{aligned} \tag{7}$$

In the mass basis the T_3 part of the down type quarks is modified. For the left handed part we have

$$-\mathcal{L}_Z = \frac{-g}{2\cos\theta_W}\begin{pmatrix}\bar{d}_L & \bar{s}_L\end{pmatrix}\begin{pmatrix}\cos^2\theta_L & \cos\theta_L\sin\theta_L \\ \cos\theta_L\sin\theta_L & \sin^2\theta_L\end{pmatrix}\begin{pmatrix}d_L \\ s_L\end{pmatrix} \tag{8}$$

For the right handed part we have

$$-\mathcal{L}_Z = \frac{-g}{2\cos\theta_W}\begin{pmatrix}\bar{d}_R & \bar{s}_R\end{pmatrix}\begin{pmatrix}\cos^2\theta_R & \cos\theta_R\sin\theta_R \\ \cos\theta_R\sin\theta_R & \sin^2\theta_R\end{pmatrix}\begin{pmatrix}d_R \\ s_R\end{pmatrix} \tag{9}$$

5. Are there photon and gluons FCNC's? Support your answer by an argument based on symmetries.
Answer: The photon and gluon couplings are always flavor symmetric. This is because they are the gauge bosons of an exact local symmetry.

6. Is there FCNC from Higgs exchange?
Answer: Yes, the mass matrix and the Higgs couplings is not aligned.

7. Do you think this model can describe Nature? In particular, look at $K^+ \to \pi^+ e^- e^+$ compared to $K^+ \to \pi^0 e^- \bar{\nu}$.
Answer: No. It has too large FCNCs and no CPV.