Inflation- Cosmological Problems

Flatness Problem

Recall (with $\Lambda = 0$):

$$\frac{k}{R^2} = H^2(\Omega - 1)$$

Divide by T^2 and evaluate today:

$$\hat{k} = \frac{k}{R_0^2 T_0^2} = H_0^2(\Omega_0 - 1)/T_0^2 \qquad < 2 \times 10^{-58}$$

Represents an initial condition on the Universe

Inflation- Cosmological Problems

Horizon Problem

Causal volume V ~ t^3
but the Universe expands as $t^{2/3}$ (matter dominated)

Today's visible Universe contains (for t at recombination)

$$(\frac{t_0}{t})^3(\frac{R}{R_0})^3 = (\frac{t_0}{t}) \sim 10^5$$

different causal horizon volumes.

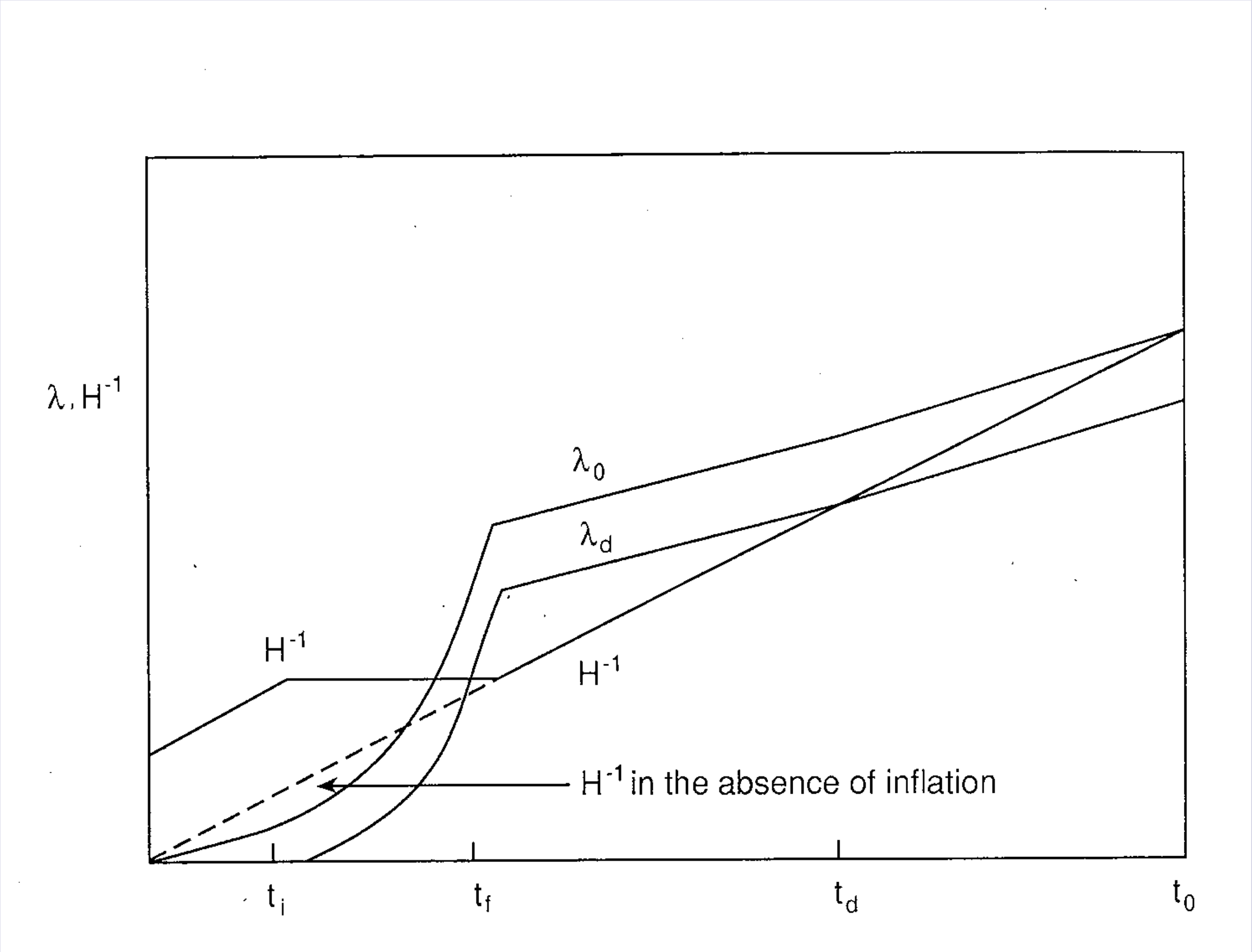
Why is

$$\frac{\Delta T}{T} \sim 10^{-5}$$

Inflation- Cosmological Problems

Perturbations Problem

Perturbations appear to have been produced outside our horizon.



Inflation- Cosmological Problems

Monopole Problem

The break-down of a GUT such as SU(5) to the SM with an explicit U(1) leads to the production of magnetic monopoles

The density of monopoles estimated by 1 per horizon volume at the time of the transition $$n_m \sim (2t_c)^{-3}$$

with $$t_c \sim 10^{-2} M_P / T_c^2$$

so $$\frac{n_m}{n_\gamma} \sim \left(\frac{10T_c}{M_P}\right)^3$$

limit: $$\frac{n_m}{n_\gamma} < O(10^{-25})$$

Inflation

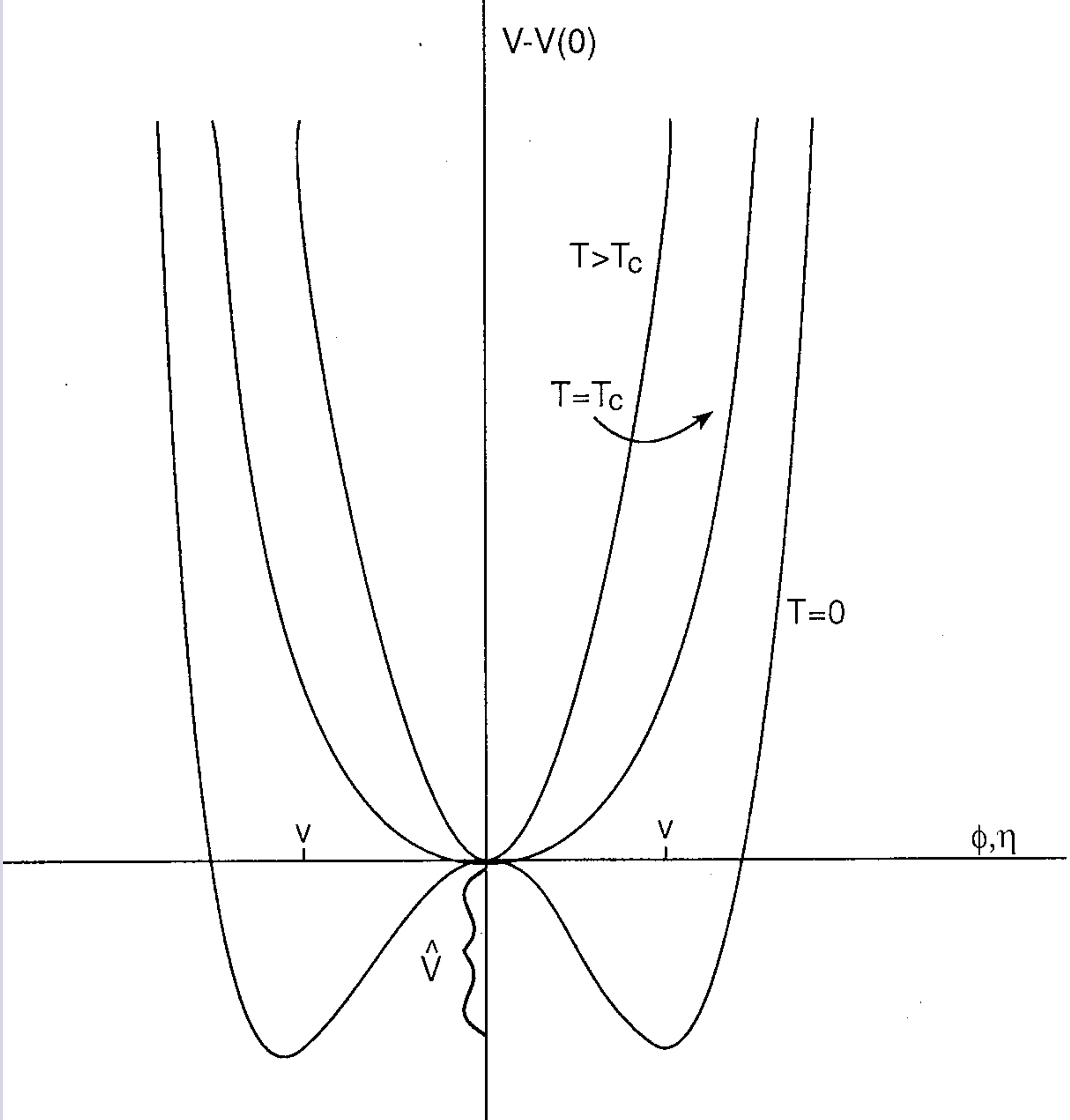
- Standard cosmology assumes an adiabatically expanding Universe, $R \sim 1/T$
- Phase transitions can violate this condition

Phase Transitions

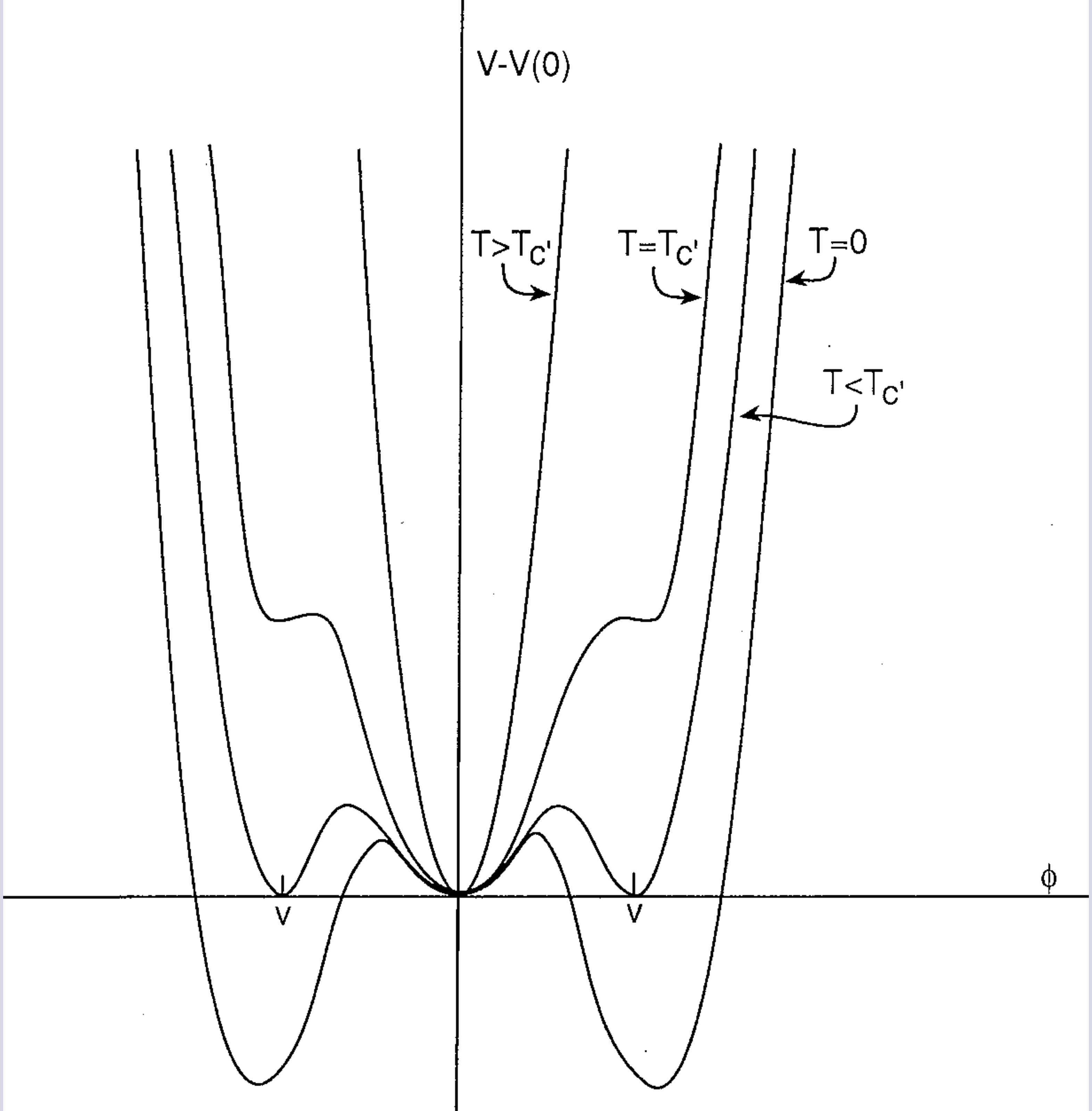
- Expect several phase transitions in the Early Universe
  - GUTS: SU5 $\rightarrow$ SU(3) x SU(2) x U(1)
  - SM: SU(2) x U(1) $\rightarrow$ U(1)
  - possibly other non-gauged symmetry breakings

- Entropy production common result

- Type of inflation will depend on the order of the phase transition



V-V(0)
T>Tc
T=Tc
T=0
v
v
φ,η
V̂

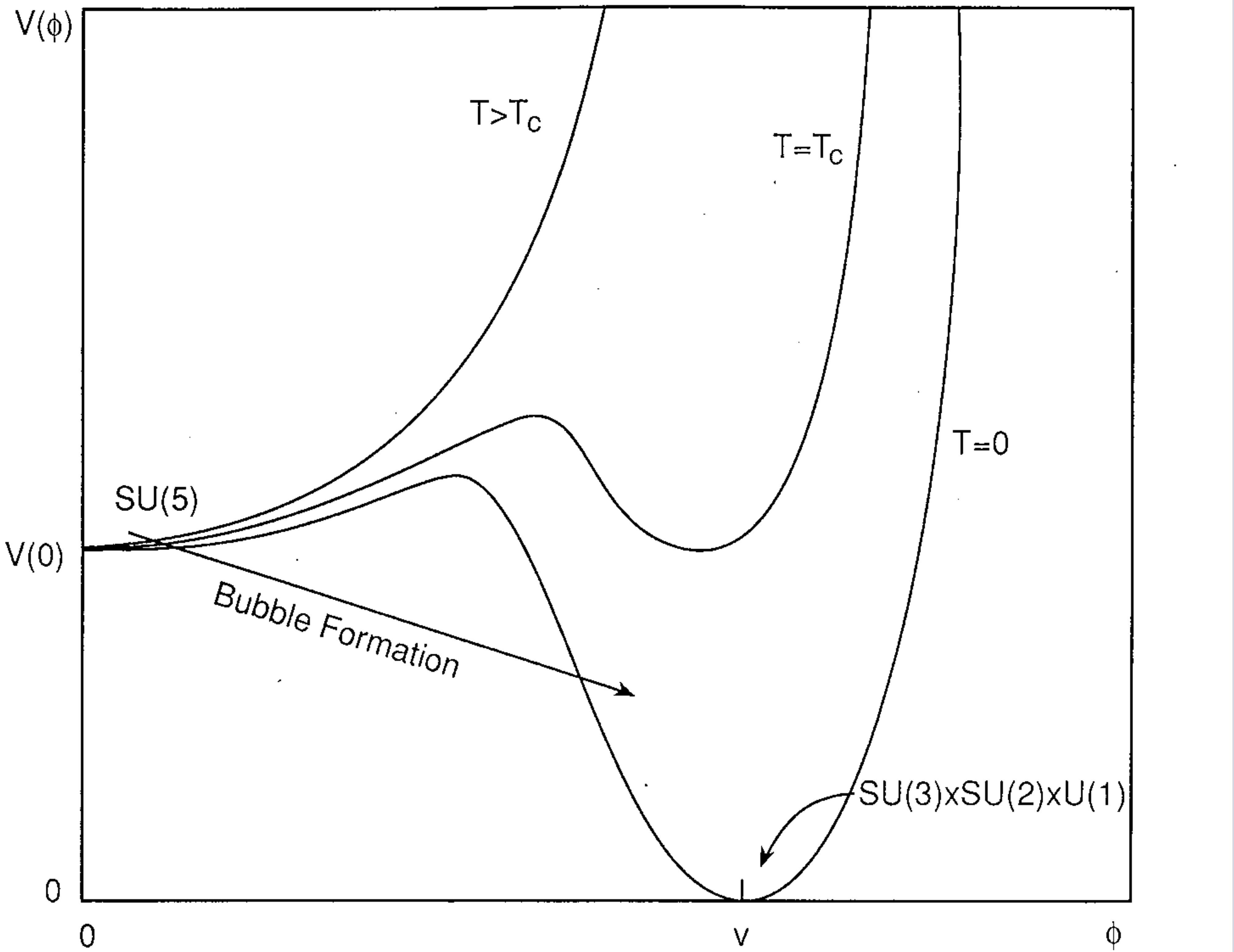

V-V(0)
T>T_C'
T=T_C'
T=0
T<T_C'
ϕ
v
v

Inflation

- Standard cosmology assumes an adiabatically expanding Universe, $R \sim 1/T$
- Phase transitions can violate this condition

Old Inflation

- Based on GUT symmetry breaking with a strong 1st order transition


V(ϕ)
T>T$_c$
T=T$_c$
T=0
SU(5)
V(0)
Bubble Formation
SU(3)xSU(2)xU(1)
0
0
v
ϕ

Inflation

- Standard cosmology assumes an adiabatically expanding Universe, $R \sim 1/T$
- Phase transitions can violate this condition

Old Inflation

- Based on GUT symmetry breaking with a strong 1st order transition
- Universe becomes trapped in false vacuum
- Vacuum energy density act as a cosmological constant
- Transition proceeds by tunneling and bubble formation

Inflation

$$\Lambda = 8\,\pi\, G_N\, V_0$$

For $\varrho \ll V_0$,

$$H^2 = \frac{\dot{R}^2}{R^2} \approx \frac{8\pi G_N V_0}{3} = \frac{\Lambda}{3}$$

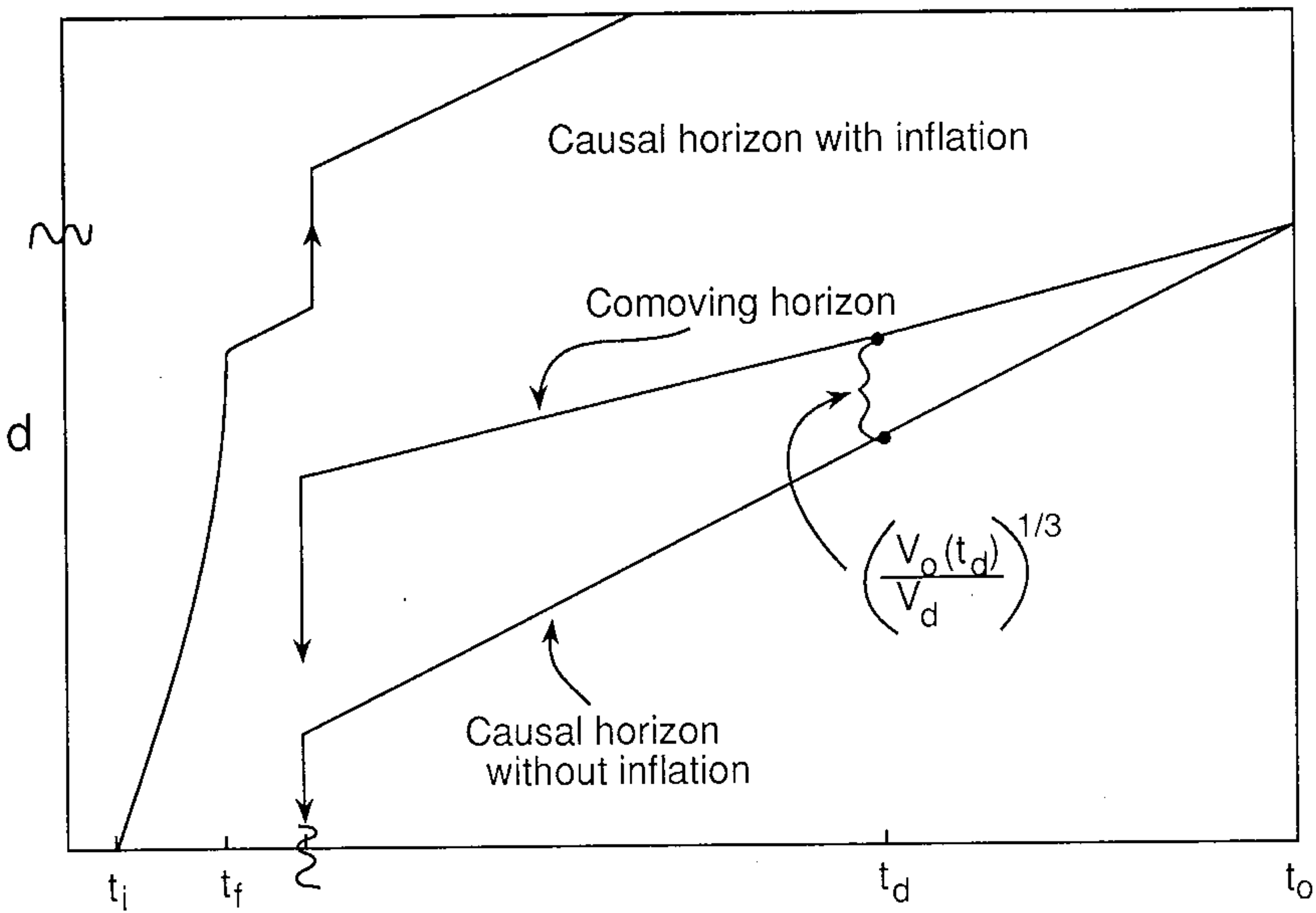
or

$$\frac{\dot{R}}{R} \approx \sqrt{\frac{\Lambda}{3}}\,; \qquad R \sim e^{Ht}$$

For $H\tau > 65$, curvature problem solved

When the transition is over, the Universe reheats to $T < V_0^{1/4} \sim T_i$, but $R \gg R_i$

Problem: Transition never completes


Causal horizon with inflation
Comoving horizon
d
$\left(\frac{V_o(t_d)}{V_d}\right)^{1/3}$
Causal horizon
without inflation
t_i
t_f
t_d
t_o

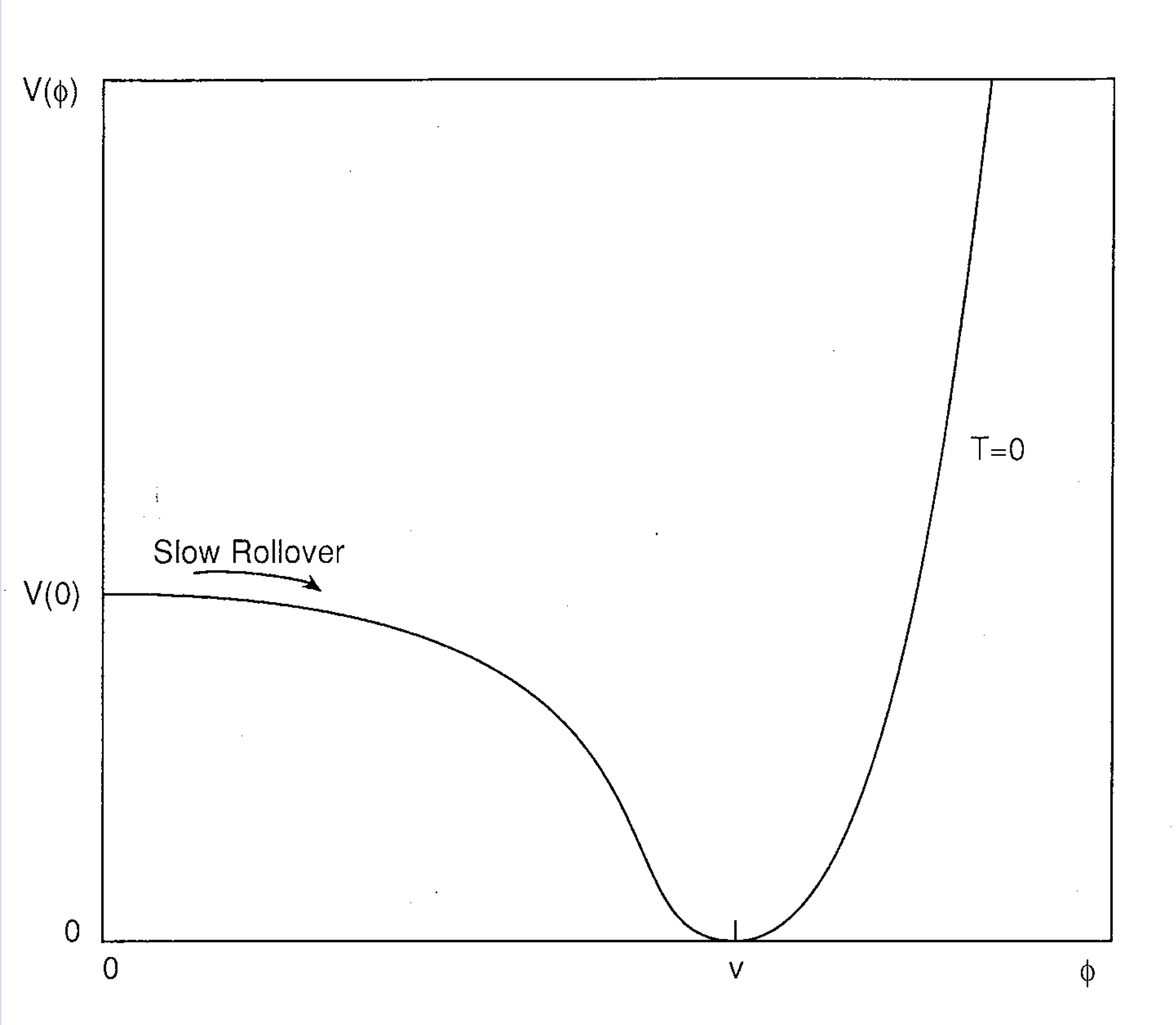
New Inflation

GUT transition a la Coleman-Weinberg

$$V(\phi) = A\phi^4(\ln\frac{\phi^2}{v^2} - \frac{1}{2}) + D\phi^2 + \hat{V}$$

$$A = \frac{1}{64\pi^2 v^4}(\sum_B g_B m_B^4 - \sum_F g_F m_F^4) = \frac{5625}{1025\pi^2 g^4}$$

New Inflation



Scalar Field Dynamics

Equations of motion

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} \simeq \ddot{\phi} + 3H\dot{\phi} + m^2(\phi)\phi = 0$$

For $|m^2| << H^2$

$$\phi \sim e^{|m^2|t/3H}$$

Field moves very little for a period

$$\tau \sim 3H/|m^2|$$

Late time evolution

$$\phi \sim \frac{v}{mt} \sin mt$$

Reheating through particle decay

$$T_R \sim (\Gamma_D M_P)^{1/2}$$

Slow Roll Conditions

Define: $$\epsilon \equiv \frac{3}{2}\left(\frac{p}{\rho}+1\right) = \frac{4\pi}{m_{\rm Pl}^2}\left(\frac{\dot\phi}{H}\right)^2 \simeq \frac{m_{\rm Pl}^2}{16\pi}\left(\frac{V'(\phi)}{V(\phi)}\right)^2$$

and: $$\eta \equiv -\frac{\ddot\phi}{H\dot\phi} \simeq \frac{m_{\rm Pl}^2}{8\pi}\frac{V''(\phi)}{V(\phi)}$$

with: $$H^2 = \left(\frac{\dot a}{a}\right)^2 = \frac{8\pi}{3m_{\rm Pl}^2}\left[\frac{1}{2}\dot\phi^2 + V(\phi)\right]$$

Require: ε and η to be small

# of e-folds: $$N = -\int H dt = -\int \frac{H}{\dot\phi}d\phi = \frac{2\sqrt{\pi}}{m_{\rm Pl}}\int\frac{d\phi}{\sqrt{\epsilon}}$$

Density Fluctuations

During the slow-roll, density fluctuations are produced

$$\frac{\delta\rho}{\rho} = 4H\delta\tau = \frac{H^2}{\pi^{3/2}\dot{\phi}} = (\frac{8\lambda}{3\pi^2})^{1/2}\ln^{3/2}(Hk^{-1})$$

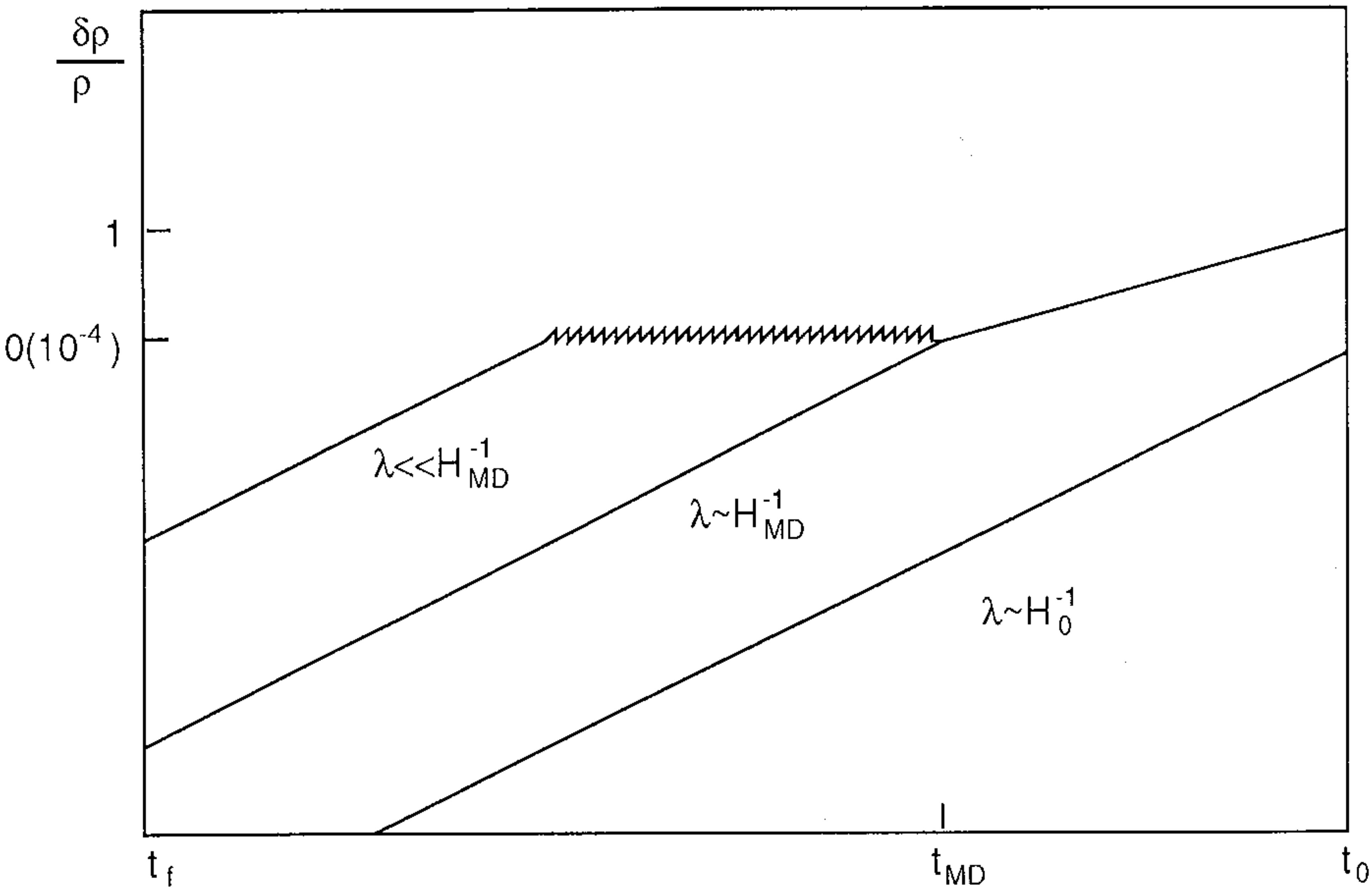
Guth & Pi

$$\propto \Delta(k)$$

Power spectrum $\Delta^2(k) = \Delta^2(k_*)(\frac{k}{k_*})^{n-1}$

$$n \simeq 1 - 6\epsilon + 2\eta \qquad n_{grav} \simeq -2\epsilon$$

$$r \simeq \frac{\Delta^2_{grav}(k_*)}{\Delta^2(k_*)} \simeq 16\epsilon$$



$\frac{\delta\rho}{\rho}$
1
$0(10^{-4})$
$\lambda << H_{MD}^{-1}$
$\lambda \sim H_{MD}^{-1}$
$\lambda \sim H_0^{-1}$
t_f
t_{MD}
t_0

Density Fluctuations

During the slow-roll, density fluctuations are produced

$$\frac{\delta\rho}{\rho} = 4H\delta\tau = \frac{H^2}{\pi^{3/2}\dot{\phi}} = (\frac{8\lambda}{3\pi^2})^{1/2}\ln^{3/2}(Hk^{-1})$$

Kill models of new inflation based on SU(5) symmetry breaking

$\Rightarrow$ Inception of the Inflaton

Many new models of inflation become possible
- Primordial, chaotic, hybrid, natural, R2, eternal, stochastic, power-law, KK, assisted, ...

Generic Inflation

$$V(\phi) = \mu^4 V(\phi/M_P)$$

Density fluctuations are roughly:

$$\frac{\delta\rho}{\rho} \sim \mathcal{O}(100)\,\frac{\mu^2}{M_P^2}$$

which can be used to fix μ

$$\frac{\mu^2}{M_P^2} \sim few \times 10^{-8}$$

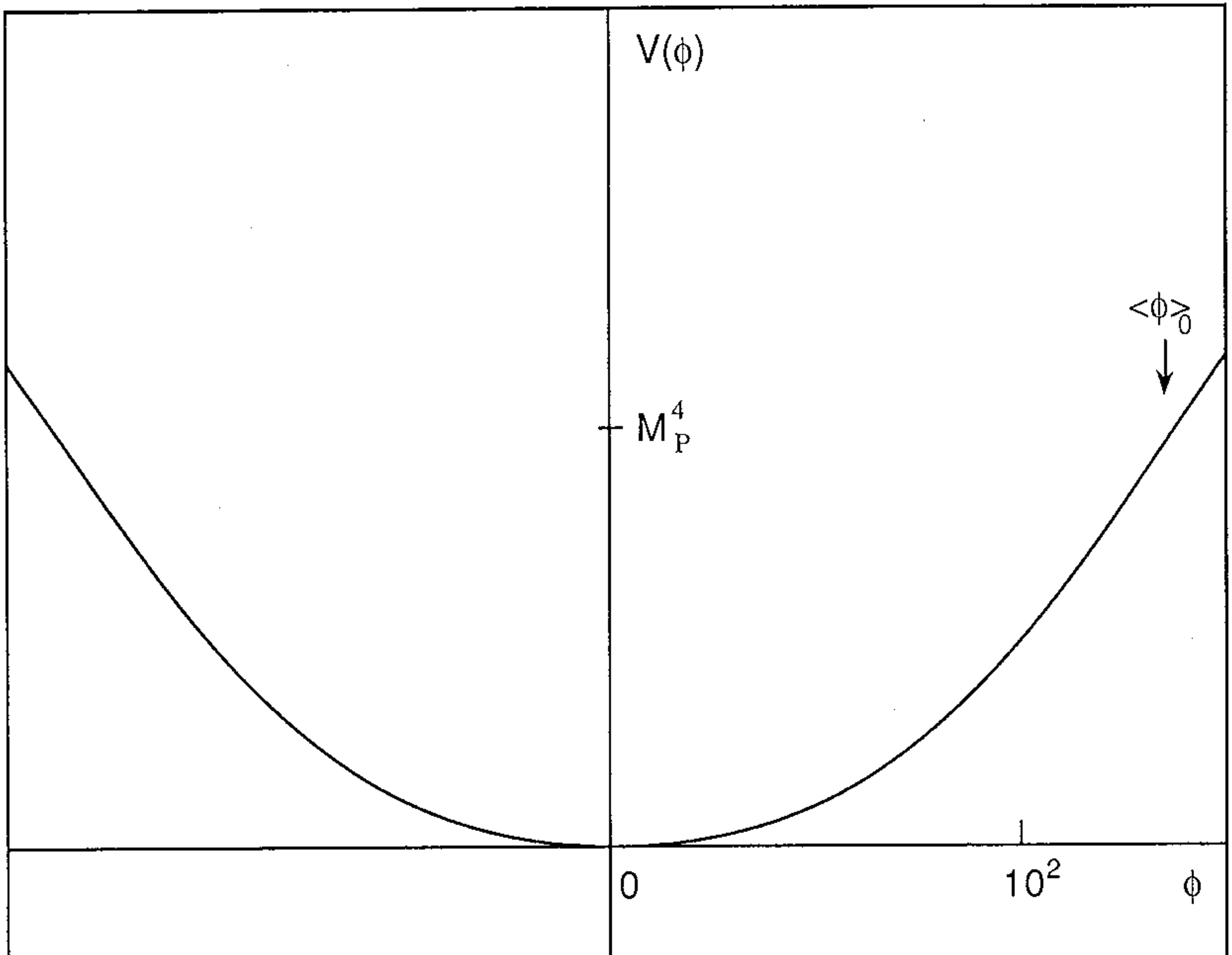
which in turns determines the Hubble parameter during inflation, the duration of inflation, and the reheat temperature.

Chaotic Inflation

Very simple potentials of the form:

$$V(\phi) = m^2\phi^2 \qquad \text{or} \qquad V(\phi) = \lambda\phi^4$$

Chaotic Inflation



Chaotic Inflation

Very simple potentials of the form:

$$V(\phi) = m^2\phi^2$$

or

$$V(\phi) = \lambda\phi^4$$

$$\epsilon = 1/120 \qquad \epsilon = 1/60$$

$$\eta = 1/120 \qquad \eta = 1/40$$

$$n = .97 \qquad n = .95$$

$$r = .13 \qquad r = .27$$

WMAP constraints on inflationary models

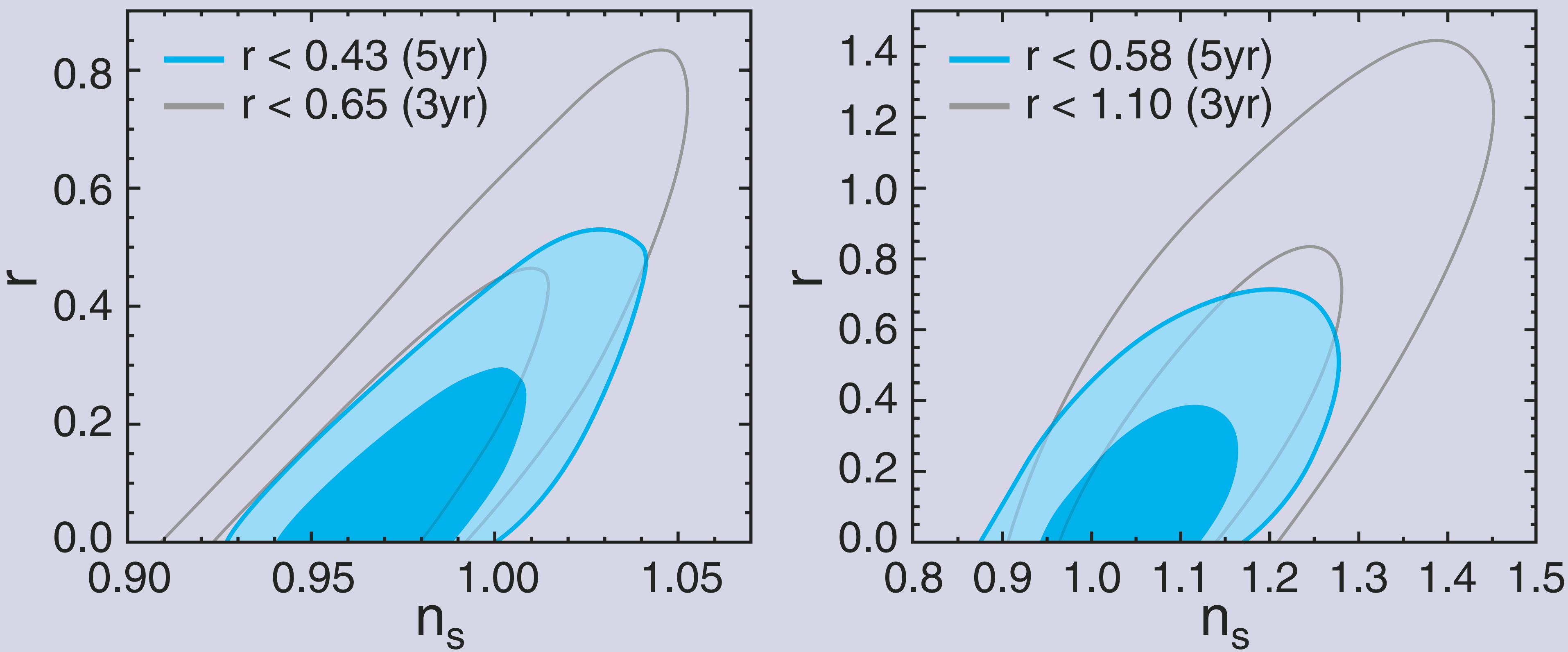


Fig. 10.— Two-dimensional marginalized constraints (68% and 95% confidence levels) on inflationary parameters r, the tensor-to-scalar ratio, and n_s, the spectral index of fluctuations, defined at $k_0 = 0.002/\mathrm{Mpc}$. One-dimensional 95% upper limits on r are given in the legend. Left: The five-year *WMAP* data places stronger limits on r (shown in blue) than three-year data (grey). This excludes some inflationary models including $\lambda\phi^4$ monomial inflaton models with $r \sim 0.27$, $n_s \sim 0.95$ for 60 e-folds of inflation. Right: For models with a possible running spectral index, r is now more tightly constrained due to measurements of the third acoustic peak. Note: the two-dimensional 95% limits correspond to $\Delta(2\ln L) \sim 6$, so the curves intersect the $r = 0$ line at the $\sim 2.5\sigma$ limits of the marginalized n_s distribution.

Anti-matter in the Universe

- On Earth?
- On the Moon?
- In the Solar System?
- In the Galaxy?
  - in cosmic rays antimatter is secondary
  - antiHelium - never observed

$$\bar{H}e = \bar{p}\bar{p}\bar{n}\bar{n}$$

- Anywhere?

Baryogenesis

The Baryon asymmetry

- Goal: To calculate η from microphysics

- Problem: In baryon symmetric universe the baryon density is determined by freeze-out of annihilations

$$\frac{n_B}{n_\gamma} = \frac{n_{\bar{B}}}{n_\gamma}$$

For T >> m_N, $$\frac{n_B}{n_\gamma} \sim O(1)$$

For T < m_N, $$\frac{n_B}{n_\gamma} \sim \left(\frac{m_N}{T}\right)^{3/2} e^{-m_N/T}$$

Baryogenesis

The Baryon asymmetry

Compute Freeze-out

Annihilations: $$\sigma v \sim \frac{1}{m_\pi^2}$$

Rate: $$\Gamma = n\sigma v \sim \frac{m_N^{3/2} T^{3/2}}{m_\pi^2} e^{-m_N/T}$$

Compare to expansion rate: $$H \sim \frac{T^2}{M_P}$$

Freeze-out at $T/m_N \sim 1/45$

$$\frac{n_B}{n_\gamma} = \frac{n_{\bar{B}}}{n_\gamma} \sim 10^{-19}$$

The Sakharov Conditions

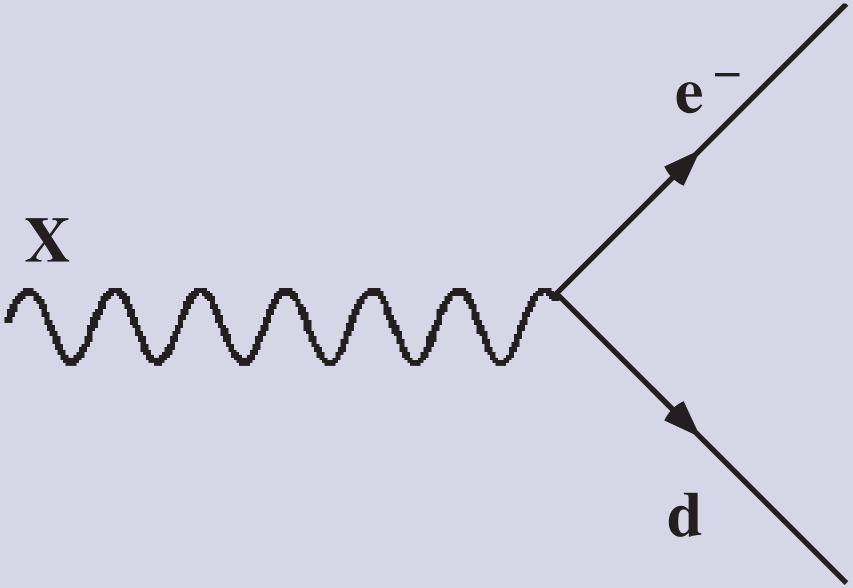
To generate an asymmetry:

1. Baryon Number Violating Interactions
2. C and CP Violation
3. Departure from Thermal equilibrium

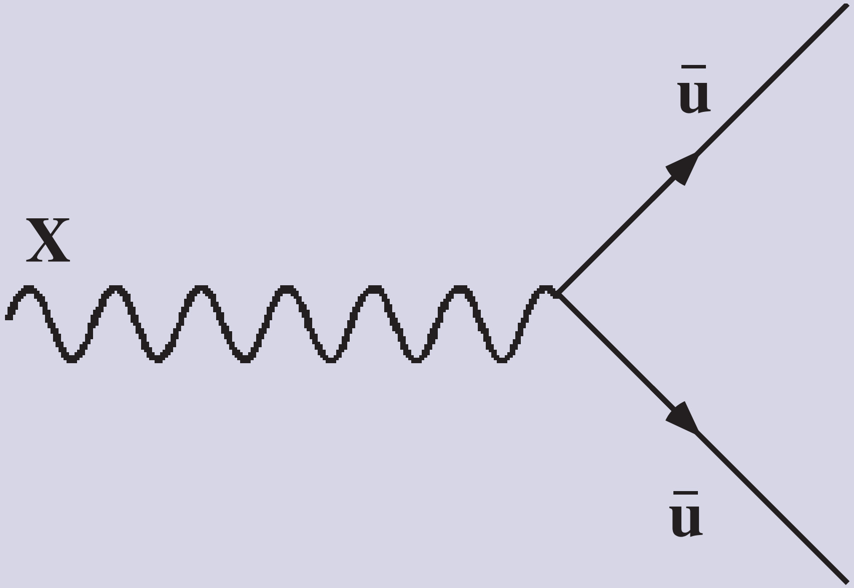
1. and 2. are contained in GUTs
3. is obtained in an expanding Universe

Grand Unified Theories

In SU(5), there are gauge (and Higgs) bosons which mediate baryon number violation. Eg.,



$\Delta B = +\ 1/3$



$\Delta B = -\ 2/3$

Out-of-equilibrium decay

Decay rate: $\Gamma \simeq \alpha M_X$

But decays occur only when $\Gamma > H$

$$\alpha M_X > N(T)^{1/2} T^2 / M_P$$

or

$$T^2 < \alpha M_X M_P N(T)^{-1/2}.$$

Out-of-equilibrium if $\Gamma < H$ at $T \sim M_X$

Require $M_X > \alpha M_P (N(M_X))^{-1/2}$

Out-of-equilibrium decay

Denote

$$\begin{aligned}
&\text{Under CPT}: && \mathit{\Gamma}(X \to 1\uparrow) = \mathit{\Gamma}(\bar{1}\downarrow \to \bar{X}) \\
&\text{Under CP}: && \mathit{\Gamma}(X \to 1\uparrow) = \mathit{\Gamma}(\bar{X} \to \bar{1}\downarrow) \\
&\text{Under C}: && \mathit{\Gamma}(X \to 1\uparrow) = \mathit{\Gamma}(\bar{X} \to \bar{1}\uparrow)
\end{aligned}$$

and let

$$r = \mathit{\Gamma}(X \to 1\uparrow) + \mathit{\Gamma}(X \to 1\downarrow)$$
$$\bar{r} = \mathit{\Gamma}(\bar{X} \to \bar{1}\uparrow) + \mathit{\Gamma}(\bar{X} \to \bar{1}\downarrow)$$

The total baryon asymmetry produced by a pair is:

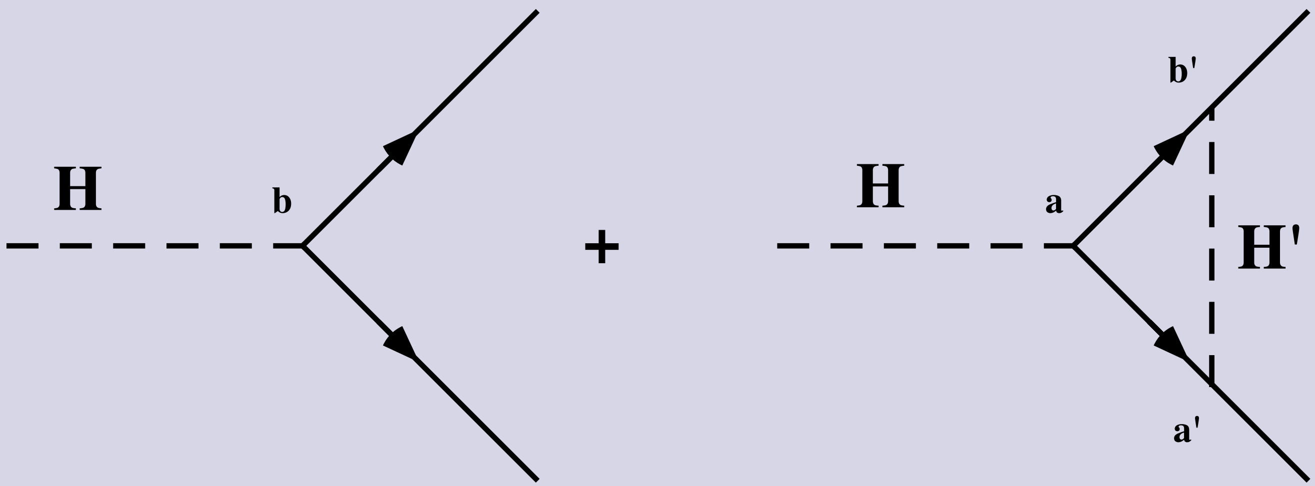
$$\begin{aligned}
\mathit{\Delta}B &= -\frac{2}{3}r + \frac{1}{3}(1-r) + \frac{2}{3}\bar{r} - \frac{1}{3}(1-\bar{r}) \\
&= \bar{r} - r = \mathit{\Gamma}(\bar{X} \to \bar{1}\uparrow) + \mathit{\Gamma}(\bar{X} \to \bar{1}\downarrow) - \mathit{\Gamma}(X \to 1\uparrow) - \mathit{\Gamma}(X \to 1\downarrow)
\end{aligned}$$

The final asymmetry becomes:

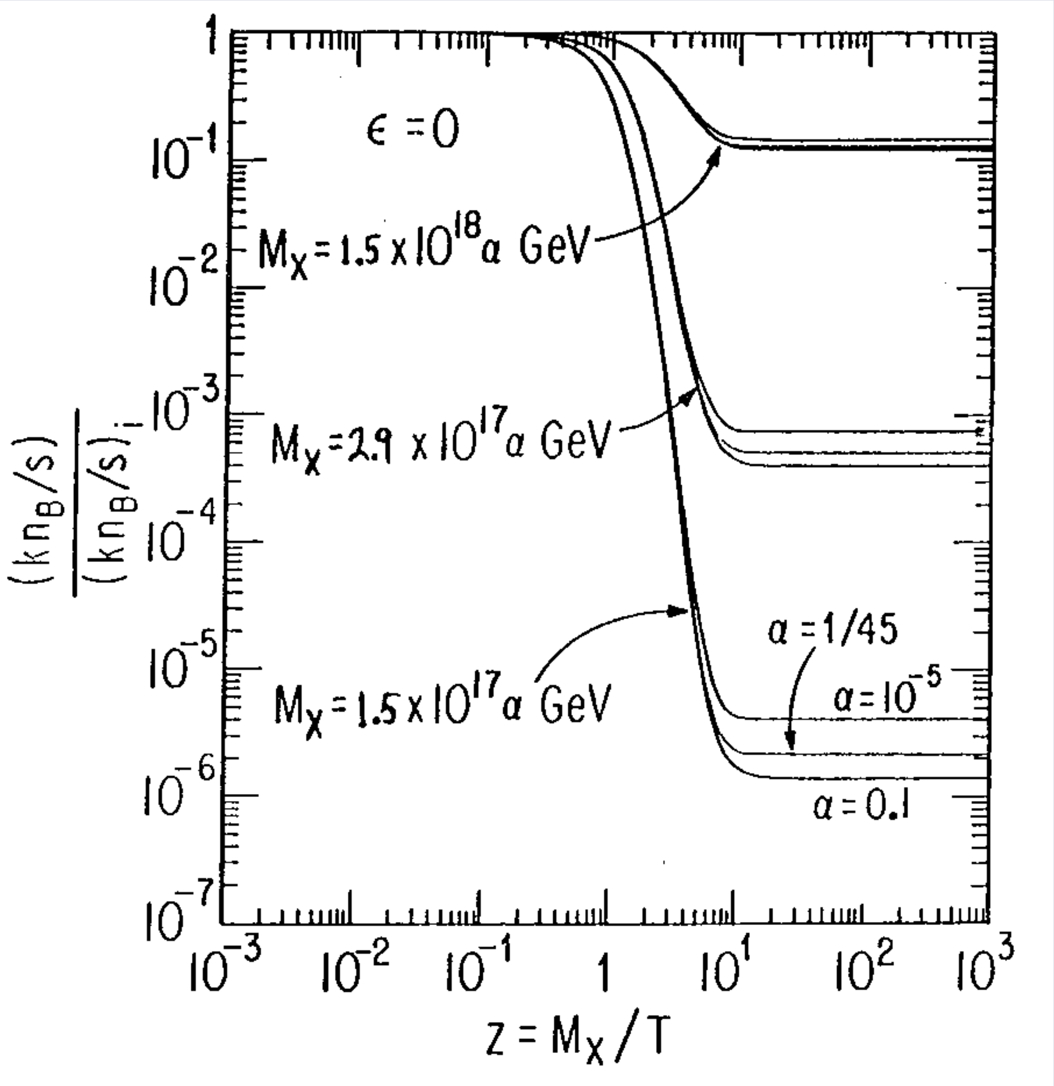
$$\frac{n_B}{s} = \frac{(\Delta B)n_X}{s} \sim \frac{(\Delta B)n_X}{N(T)n_\gamma} \sim 10^{-2}(\Delta B)$$

where $\Delta B = (\bar{r} - r)$.

But require something like:

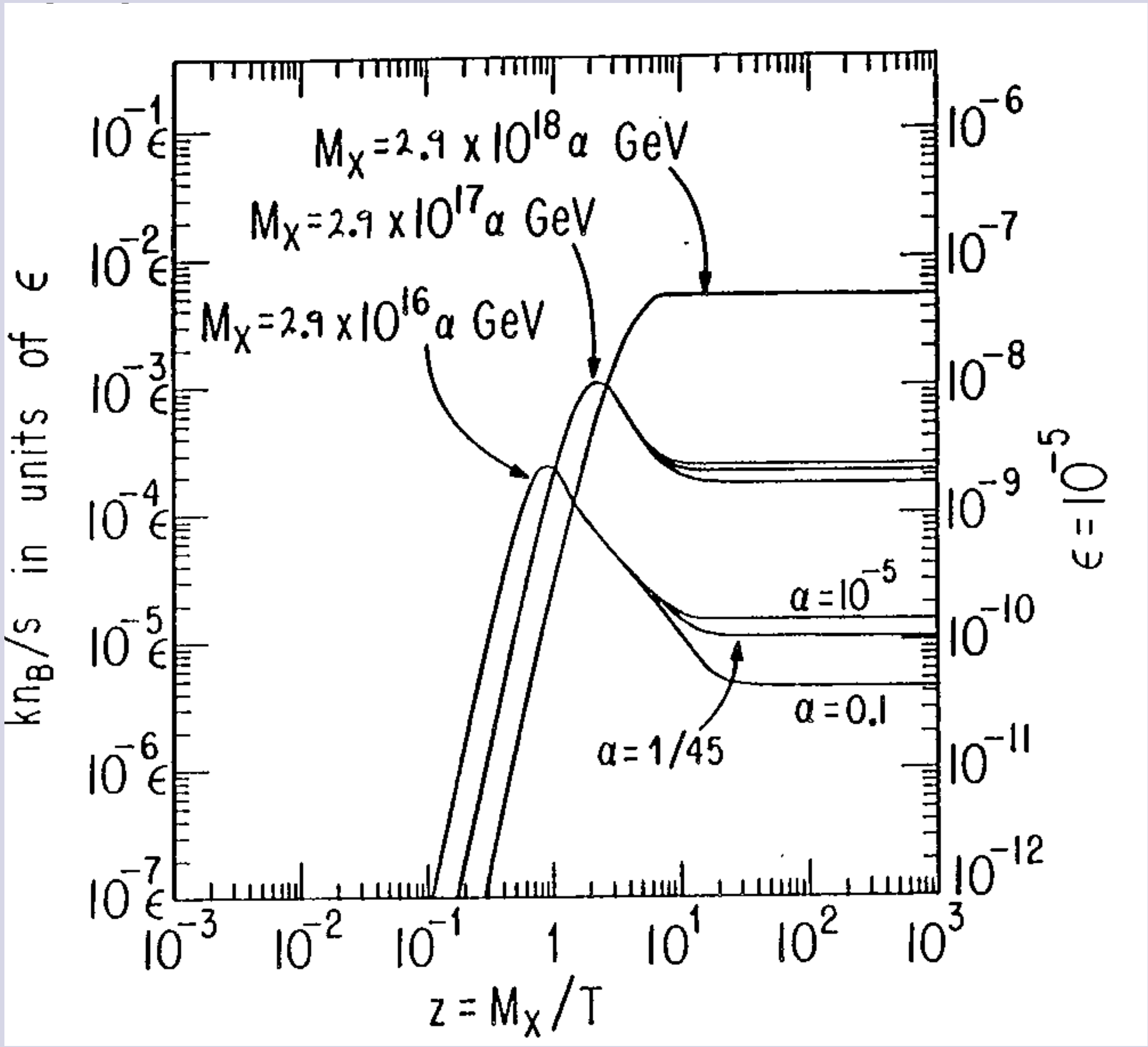


Damping of initial asymmetries



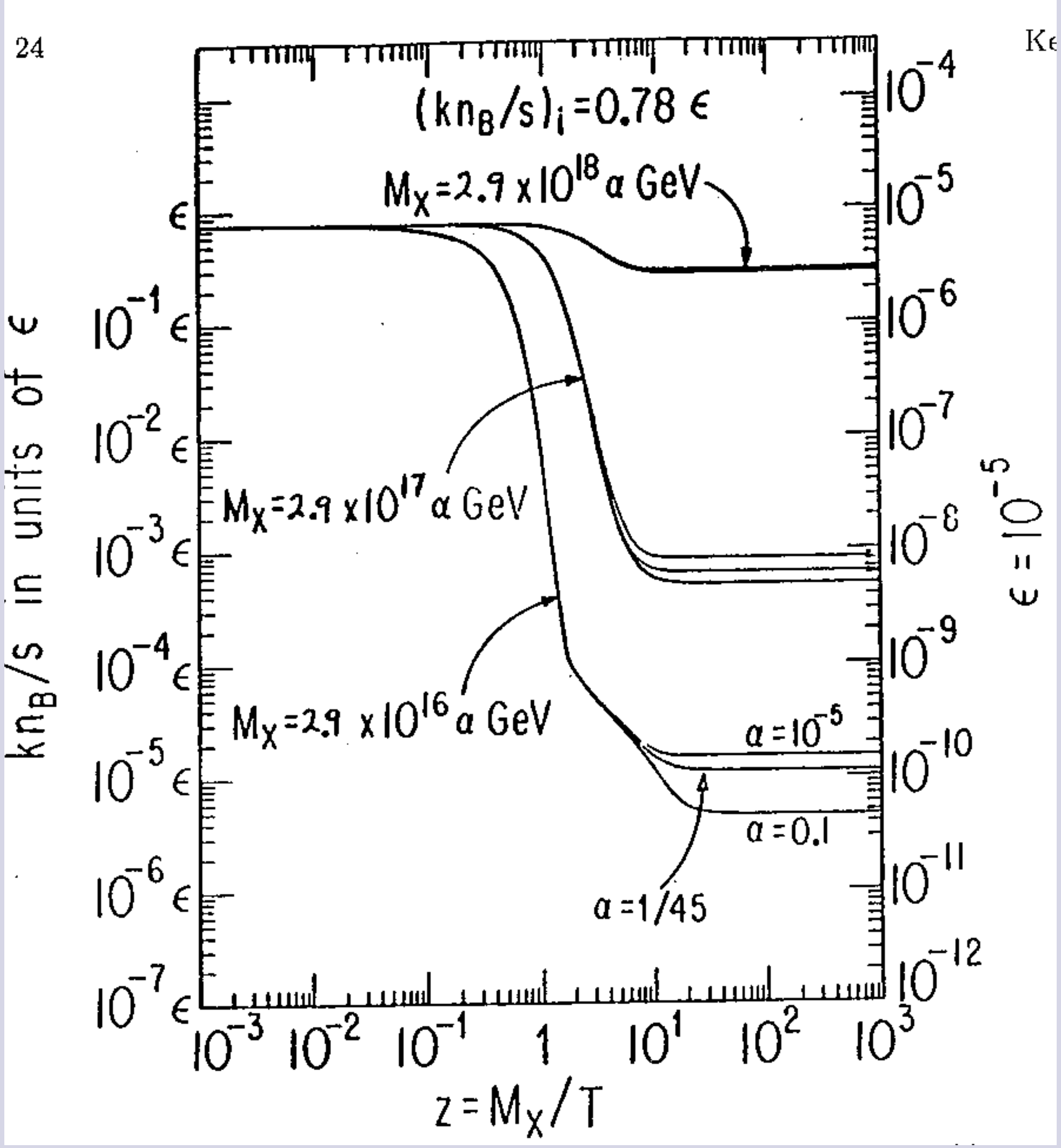
Fry et al.

Generation of an asymmetry



Fry et al.

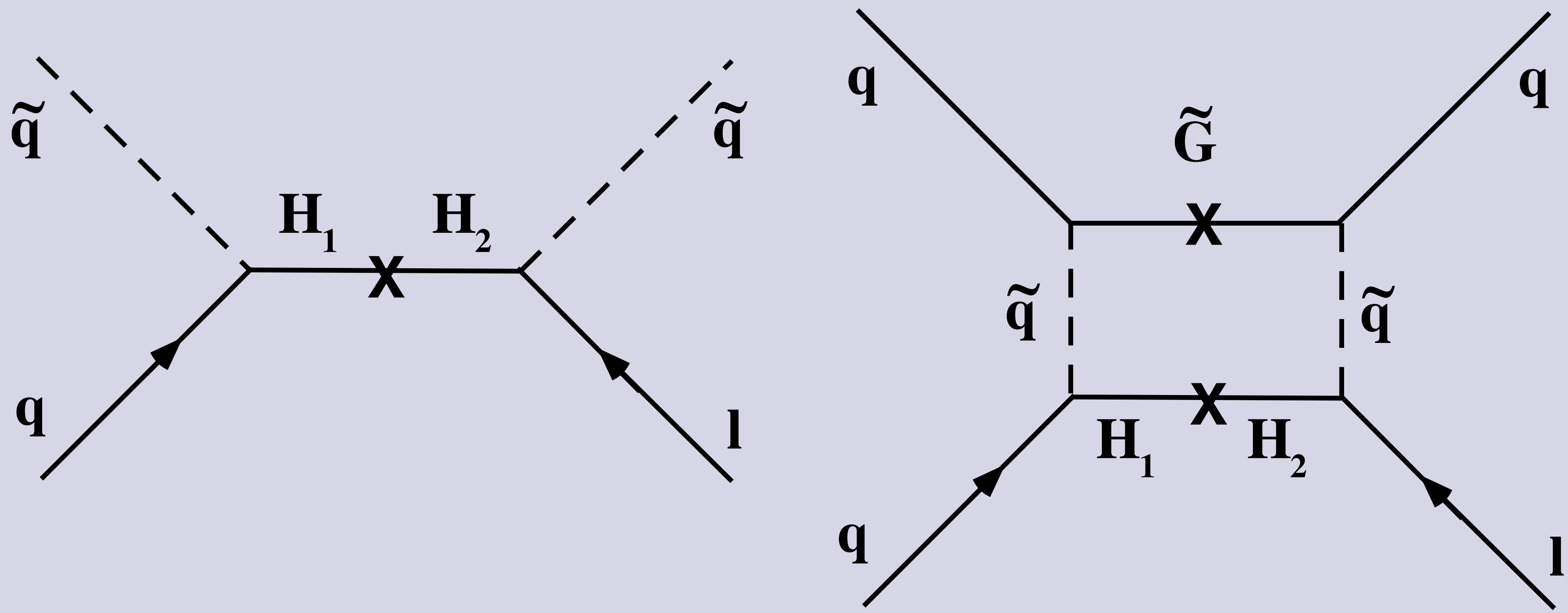
Final asymmetry



Fry et al.

Supersymmetry

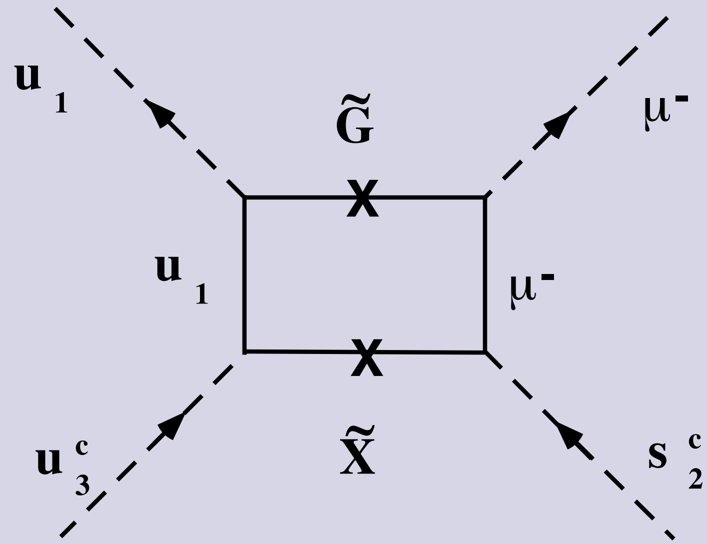
New baryon number violating operators



Affleck-Dine baryogenesis

Utilize F- and D- flat directions

$$u_3^c = a \qquad s_2^c = a \qquad -u_1 = v \qquad \mu^- = v \qquad b_1^c = e^{i\phi}\sqrt{v^2 + a^2}$$

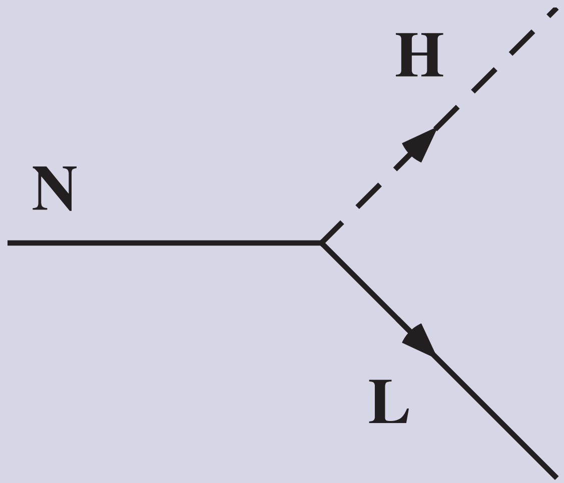


$$V(\phi) = \tilde{m}^2 \phi \phi^* + \frac{1}{2} i\lambda(\phi^4 - \phi^{*4})$$

Leptogenesis

Consider extension to SM with right-handed neutrinos and a see-saw mechanism

Can generate a lepton asymmetry from out-of-equilibrium decays of N



Sphaleron interactions to convert lepton asymmetry to a baryon asymmetry

$$B = \frac{28}{79}\left(B - L\right)$$