Flavor physics

Yuval Grossman

Cornell

Yesterday...

- What is HEP
- The SM, and its flavor structure
- CKM matrix

Today we will talk about counting parameters and how to determining the flavor parameters

Parameter counting

How many parameters we have?

How many parameters are physical?

- “Unphysical” parameters are those that can be set to zero by a basis rotation
- General theorem

$$N(\text{Phys}) = N(\text{tot}) - N(\text{broken})$$

  - $N(\text{Phys})$, number of physical parameters
  - $N(\text{tot})$, total number of parameters
  - $N(\text{broken})$, number of broken generators

Example: Zeeman effect

A hydrogen atom with weak magnetic field

- The magnetic field add one new physical parameter, B

$$V(r) = \frac{-e^2}{r} \qquad V(r) = \frac{-e^2}{r} + B_x\hat{x} + B_y\hat{y} + B_z\hat{z}$$

- But there are 3 total new parameters
- The magnetic field breaks explicitly: $SO(3) \to SO(2)$
- 2 broken generators, can be "used" to define the z axis

$$N(\text{Phys}) = N(\text{tot}) - N(\text{broken}) \quad \Rightarrow \quad 1 = 3 - 2$$

Back to the flavor sector

Without the Yukawa interaction, a model with N copies of the same field has a $U(N)$ global symmetry

- It is just the symmetry of the kinetic term

$$\mathcal{L} = \bar{\psi}_i D_\mu \gamma^\mu \psi_i, \qquad i = 1, 2, ..., N$$

- $U(N)$ is the general rotation in N dimensional complex space
- $U(N) = SU(N) \times U(1)$ and it has N^2 generators

Lepton sector

First example, leptons in the “old” SM: L, E

- One Yukawa matrix: $Y\bar{L}\phi E$, $N_T = 18$
- Global symmetries: $U(3)_E \times U(3)_L$, 18 generators
- Exact accidental symmetries: $U(1)_e \times U(1)_\mu \times U(1)_\tau$, 3 generators
- Broken generators due to the Yukawa: $N_B = 18 - 3 = 15$
- Physical parameters: $N_P = 18 - 15 = 3$. They are the 3 charged lepton masses

Two generations of quarks

Q_L^i, D_R^i, U_R^i, $i = 1, 2$

Do it yourself...

$$N_T =$$
$$N_G =$$
$$N_U =$$
$$N_B = N_G - N_U =$$

So we have

$$N_P = N_T - N_B =$$

how many are masses and how many are mixing angles?

The SM flavor sector

Back to the SM with three generations

- $N_T = 2 \times 18 = 36$
- $N_G = 3 \times 9 = 27$
- $N_U = 1$
- $N_B = 27 - 1 = 26$
- $N_P = 36 - 26 = 10$
- 6 quark masses, 3 mixing angles and one CPV phase

Remark: The broken generators are 17 Im and 9 Re. We have 18 to “start with” so the physical ones are $18 - 17 = 1$ and $18 - 9 = 9$

Homework

Consider a model with the same gauge symmetry and SSB as in the SM. The fermions, however, are

$$Q_L(3,2)_{1/6}, \quad S_L(3,1)_{-1/3}, \quad Q_R(3,2)_{1/6}, \quad S_R(3,1)_{-1/3}$$

- What is the spectrum of this model? That is, what are the quarks after SSB. Note that you can also have "bare masses" in this model. Also, there is no flavor index
- How many physical parameters there are, and what are they?
- Are there W exchange flavor changing interactions?
- Is there tree level FCNC in this model?

The CKM matrix

The flavor parameters

- The 6 masses. We kind of know them. There is a lot to discuss, but I will not do it in these lectures
- The CKM matrix has 4 parameters
  - 3 mixing angles (the orthogonal part of the mixing)
  - One phase (CP violating)
- We will concentrate on trying to find ways to determine the CKM three mixing angles and one phase. Here we will get into some details

The CKM matrix

$$\mathcal{L}_W = \frac{g}{\sqrt{2}} \overline{U_L} V \gamma^\mu D_L W_\mu^+ + \text{h.c.}$$

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- CKM is unitary

$$\sum V_{ij} V_{ik}^* = \delta_{jk}$$

- Experimentally, $V \sim 1$. Off diagonal terms are small
- Many ways to parametrize the matrix

CKM parametrization

- The standard parametrization

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$.

- In general there are 5 entries that carry a phase
- Experimentally: (will explain later how these measurements were done)

$$|V| \approx \begin{pmatrix} 0.97383 & 0.2272 & 3.96 \times 10^{-3} \\ 0.2271 & 0.97296 & 4.221 \times 10^{-2} \\ 8.14 \times 10^{-3} & 4.161 \times 10^{-2} & 0.99910 \end{pmatrix}$$

The Wolfenstein parametrization

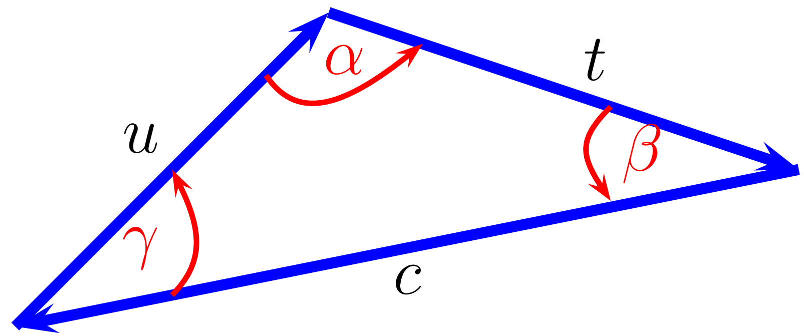
- Since $V \sim 1$ it is useful to expand it

$$V \approx \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

- One small parameter $\lambda \sim 0.2$, and three (A, ρ, η) that are roughly $O(1)$
- As always, be careful (unitarity...)
- Note that to this order only V_{13} and V_{31} have a phase

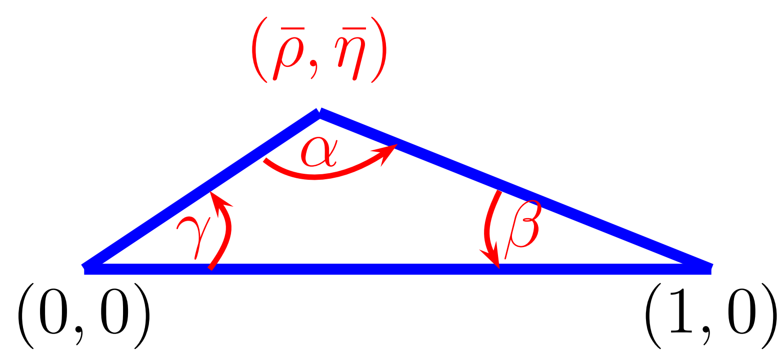
The unitarity triangle

A geometrical presentation of $V_{ub}^{*}V_{ud}+V_{tb}^{*}V_{td}+V_{cb}^{*}V_{cd}=0$



Rescale by the c size and rotated

$$A\lambda^{3}\left[(\rho+i\eta)+(1-\rho-i\eta)+(-1)\right]=0$$



CKM determination

CKM determination

- Basic idea: Measure the 4 parameters in many different ways. Any inconstancy is a signal of NP
- Problems: Experimental errors and theoretical errors
- Have to be smart...
  - Smart theory to reduce the errors
  - Smart experiment to reduce the errors
- There are cases where both errors are very small

Classifications

Two classifications:

- Parameters
  - Sides of the UT (magnitudes of CKM elements)
  - Angles of the UT (relative phases between CKM elements)
  - Combination of those
- Amplitudes
  - Tree (mostly SM)
  - Loop (SM and maybe also NP)
  - Combination of those

Experimental issues

Just very brief

- Many times we look at very small rates or small asymmetries (we like to probe small couplings). Statistics is needed
- Very important to get the PID (like K/π separation)
- Flavor tagging: is it a B or a $\bar{B}$
- CP properties: the detector is made of matter

Theoretical uncertainties

Always: QCD

- We calculate with quark, but we measure hadrons
- The strong interaction is strong. No perturbation theory. Really a problem

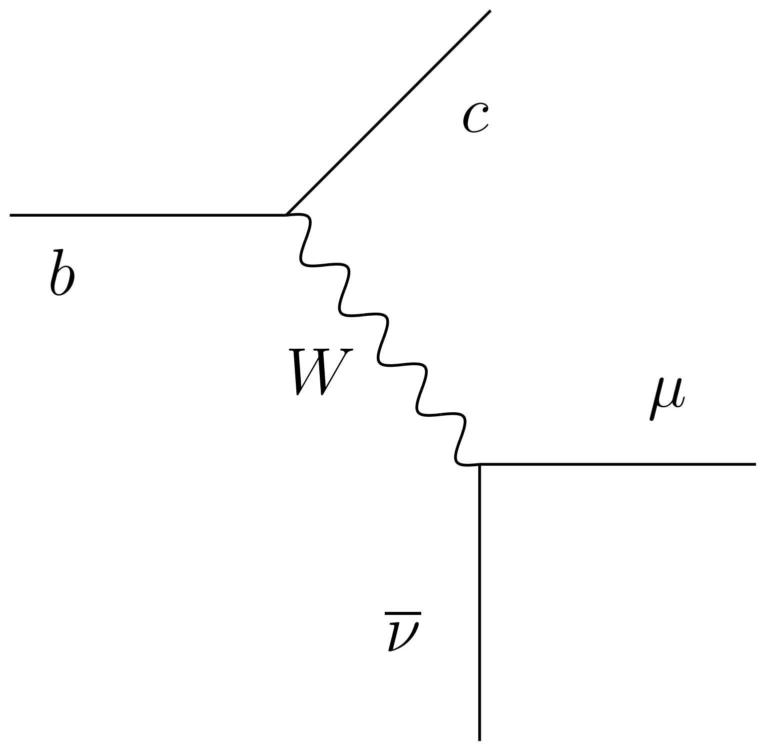
Solutions:

- We use some approximate symmetries: isospin, flavor SU(3), HQS
- There are cases where one can construct observables where the hadronic physics cancels

Measuring sides

Tree level decays are sensitive to absolute values of CKM element

$$\Gamma(B \to X_c \mu \nu) \propto |V_{cb}|^2$$



Measuring sides: problems

Not so simple...

$$\Gamma(b \to c\mu\nu) \propto m_b^5 |V_{cb}|^2$$

- Because the b is heavy, $m_b \gg \Lambda_{QCD}$ we can expand and we know that

$$\Gamma(b \to c\mu\nu) \approx \Gamma(B \to X_c\mu\nu)$$

- Not easy to get m_b the mass of the b quark. Again, we use HQS and use m_B, the B meson mass
- Using symmetries, and expanding around them we can get rather accurate determination

Always: Look for a process where we have sensitivity, and work our way around QCD

An aside: what is m_B?

$$m_B = ?$$

Other sides

Similar issues with other tree level decays

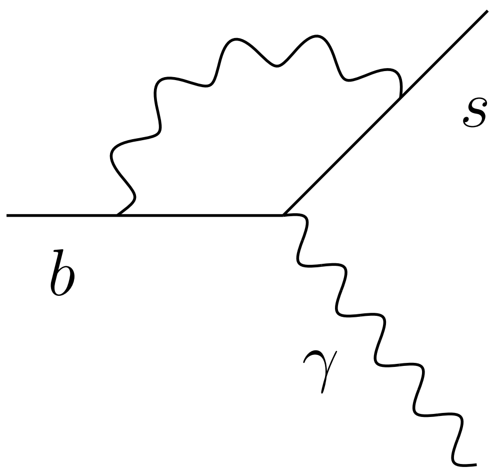
- β-decay, $d \to ue\bar{\nu} \propto V_{ud}$; Isospin
- K-decay, $s \to ue\bar{\nu} \propto V_{us}$; Isospin and SU(3)
- D-decay, $c \to qe\bar{\nu} \propto V_{cq}$ $q = d, s$; HQS
- B decays can be used also for V_{ub}. Harder
- Not easy with top. Cannot tag the final flavor, low statistics

Loop decays

- We have sensitivity to magnitude of CKM elements in loops
- More sensitive to V_{tq} that is harder to get in tree level decays
- But at the same time it may be modified by new heavy particles
- This is a general argument. NP is likely to include “heavy” particles, that can affect loop processes much more than tree level decays

Loop: example

$$A(b \to s\gamma) \propto \sum V_{ib} V_{is}^*$$



What is $\sum V_{ib} V_{is}^*$?

GIM Mechanism

what we really have is

$$A(b \to s\gamma) \propto \sum V_{ib} V_{is}^* f(m_i)$$

- Because the CKM is unitary, the m_i independent term in f vanishes
- Must be proportional to the mass (in fact, m_i^2) so the heavy fermion in the loop is dominant
- In Kaon decay this gives m_c^2/m_W^2 extra suppression. Numerically not important for b decays
- CKM unitarity and tree level Z exchange are related. (Is the diagram divergent?)

Meson mixing

Meson mixing

- The phenomena of neutral meson mixing is very important for several reasons
- General QM of a two level system

$$|f_1\rangle = |1\rangle + |2\rangle \qquad |f_2\rangle = |1\rangle - |2\rangle$$

$|f_1\rangle$ is a "flavor" ES. $|1\rangle$ is a mass ES

- Time evolution (up to overall irrelevant phase)

$$|f_1\rangle(t) = \exp\left[i\Delta E t/2\right]|1\rangle + \exp\left[-i\Delta E t/2\right]|2\rangle$$

Flavor oscillation

The probability to measure flavor f_i at time t is

$$|\langle f_1|f_1\rangle|^2 = \frac{1+\cos\Delta E t}{2}$$
$$|\langle f_1|f_2\rangle|^2 = \frac{1-\cos\Delta E t}{2}$$

- Oscillations with frequency ΔE
- In the rest frame it is just Δm
- Later we explain how, for now lets us assume we measure the flavor at production, $t=0$, and when it decays at time t

Time scales

$$|\langle f_1|f_1\rangle|^2 = \frac{1+\cos\Delta E t}{2}$$

- $\Delta E \ll t$. No oscillation, flavor is conserved
- $\Delta E \gg t$. The oscillation is averaged out. We have an "incoherent" equal sum of the two flavors
- $\Delta E \sim t$. Oscillations are important

What is the relevant times scale? It is the decay time

$$x \equiv \frac{\Delta m}{\Gamma}$$

We are sensitive to mass differences of order the width.
Can be very small $\Gamma/m \sim 10^{-14}$