Field Theory and Standard Model

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Why Quantum Field Theory?

(i) Fields: space–time aspects

- field = quantity $\phi(\vec{x}, t)$,
  defined for all points of space $\vec{x}$ and time t

  physical system defined by a Lagrangian $\mathcal{L}(\phi(\vec{x}, t))$

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  defined for all points of space $\vec{x}$ and time t

  physical system defined by a Lagrangian $\mathcal{L}(\phi(\vec{x}, t))$
- fields and Lagrangian formalism accommodate
  - symmetries:
    – space–time symmetry: Lorentz invariance
    – internal symmetries (e.g. gauge symmetries)
  - causality
  - local interactions

(ii) Particles: quantum theory aspects

- particles are classified by mass m and spin s
- quantum numbers $m, s,\ \vec{p}, s_3, charge,\ isospin, \cdots$
  = eigenvalues of observables
- particle states $|m, s;\ \vec{p}, s_3, \cdots >$
  form quantum mechanical Hilbert space

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merging space-time and quantum aspects:

Quantum Field Theory

fields are operators that create/annihilate particles

Outline

1. Elements of Quantum Field Theory
2. Cross sections and decay rates
3. Gauge Theories
    3.1. Abelian gauge theories – QED
    3.2. Non-Abelian gauge theories
    3.3. QCD
4. Higgs mechanism
    4.1. Spontaneous symmetry breaking (SSB)
    4.2. SSB in gauge theories
5. Electroweak interaction and Standard Model

5. Phenomenology of W and Z bosons, precision tests

6. Higgs bosons

Notations and Conventions

$$\mu, \nu, .. = 0, 1, 2, 3; \qquad k, l, .. = 1, 2, 3$$

$$x = (x^\mu) = (x^0, \vec{x}), \quad x^0 = t \qquad (\hbar = c = 1)$$

$$p = (p^\mu) = (p^0, \vec{p}), \quad p^0 = E = \sqrt{\vec{p}^2 + m^2}$$

$$a_\mu = g_{\mu\nu}\, a^\nu, \qquad (g_{\mu\nu}) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$a^2 = a_\mu a^\mu, \quad a \cdot b = a_\mu b^\mu = a^0 b^0 - \vec{a} \cdot \vec{b}$$

$$\partial_\mu = \frac{\partial}{\partial x^\mu} = g_{\mu\nu}\, \partial^\nu, \quad \partial^\nu = \frac{\partial}{\partial x_\nu} \qquad [\, \partial^0 = \partial_0, \;\; \partial^k = -\partial_k \,]$$

$$\Box = \partial_\mu \partial^\mu = \frac{\partial^2}{\partial t^2} - \Delta$$

1. Elements of Quantum Field Theory

Fields in the Standard Model

- spin 0 particles: scalar fields $\phi(x)$
- spin 1 particles: vector fields $A_\mu(x), \quad \mu = 0, \ldots 3$
- spin 1/2 fermions: spinor fields $\psi(x) = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix}$

Lagrangian: $\mathcal{L}(\phi, \partial_\mu \phi)$ Lorentz invariant

Action: $S = \int \mathrm{d}^4 x \, \mathcal{L}(\phi(x), \cdots)$ Lorentz invariant

Hamilton's principle: $\delta\, S = 0 \Rightarrow e.o.m.$ Lorentz covariant

free fields: $\mathcal{L}$ is quadratic in the fields $\Rightarrow$ e.o.m. are linear diff. eqs.

equations of motions from $\delta S = S[\phi + \delta\phi] - S[\phi] \,=\, 0$

$\Rightarrow$ Euler-Lagrange equations

- mechanics of particles: $L(q_1, \ldots q_n, \dot{q}_1, \ldots \dot{q}_n)$

  e.o.m. $\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} \,=\, 0$

- field theory: $q_i \to \phi(x), \quad \dot{q}_i \to \partial_\mu \phi(x)$

  e.o.m. $\partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} - \frac{\partial \mathcal{L}}{\partial \phi} \,=\, 0$ field eq.

example: scalar field $\phi(x), \quad \mathcal{L} = \frac{1}{2}\,(\partial_\mu \phi)^2 - \frac{m^2}{2}\,\phi^2$

field equation $\Box\,\phi + m^2\,\phi = 0$

solution $\phi(x) = \frac{1}{(2\pi)^{3/2}} \int \frac{\mathrm{d}^3k}{2k^0}\,[a(k)\,e^{-ikx} \,+\, a(k)^\dagger\,e^{ikx}\,]$

Scalar field [spin 0 , mass m]

neutral: $\phi = \phi^+$, $\quad \mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{m^2}{2}\phi^2$

charged: $\phi \neq \phi^+$, $\quad \mathcal{L} = (\partial_\mu \phi)^+ (\partial^\mu \phi) - m^2 \phi^+ \phi$

$$\delta S = 0 \xrightarrow{\text{e.o.m.}} \boxed{(\Box + m^2)\phi = 0} \quad \text{Klein-Gordon-Eq.}$$

Propagator (= Green's function) : $D(x-y)$

consider point-like source at y:

$$(\Box + m^2) D(x-y) = \delta^4(x-y) \xrightarrow{\text{Fourier-Transf.}} (-k^2+m^2)D(k) = 1$$

$$\boxed{iD(k) = \frac{i}{k^2 - m^2 + i\varepsilon}}$$

$x^0 > y^0$:	particle $y \to x$
$y^0 > x^0$:	anti-particle $x \to y$

causality

arrow = flow of particle charge

Vector field [Spin 1, mass $m \neq 0$] "massive photon"

$A_\mu(x)$ $(\mu = 0, \ldots 3)$, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ field strength tensor

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} m^2 A_\mu A^\mu$$

$$\delta S = 0 \xrightarrow{\text{e.o.m}} \underbrace{\left[(\Box + m^2) g^{\mu\nu} - \partial^\mu \partial^\nu \right]}_{=: K^{\mu\nu}} A_\nu = 0$$

solutions: $\sim \varepsilon_\mu e^{\pm ikx}$, with $\varepsilon \cdot k = 0$, $\varepsilon^2 = -1$

3 independent polarization vectors $\varepsilon_\mu^{(\lambda)}(k)$ $(\lambda = 1, 2, 3)$

polarization sum:

$$\sum_{\lambda=1}^{3} \varepsilon_\mu^{(\lambda)} \varepsilon_\nu^{(\lambda)} = -g_{\mu\nu} + \frac{k_\mu k_\nu}{m^2}$$

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- propagator = Green's function $D_{\mu\nu}(x-y)$

  propagation of field from point-like source at y

$$K^{\mu\rho} D_{\rho\nu}(x-y) = g^{\mu}{}_{\nu}\, \delta^4(x-y)$$

Fourier transformation: $D_{\mu\nu}(x-y) = \int \frac{d^4k}{(2\pi)^4} D_{\mu\nu}(k) e^{ik(x-y)}$

$$K^{\mu\nu} = (\Box + m^2) g^{\mu\nu} - \partial^\mu \partial^\nu \longrightarrow (-k^2+m^2) g^{\mu\nu} + k^\mu k^\nu$$

→ algebraic equation: $\left[(-k^2+m^2) g^{\mu\rho} + k^\mu k^\rho\right] D_{\rho\nu}(k) = g^{\mu}{}_{\nu}$

solution [with $+i\varepsilon$ convention → causality]

$$\boxed{i D_{\rho\nu}(k) = \frac{i}{k^2 - m^2 + i\varepsilon}\left(-g_{\rho\nu} + \frac{k_\rho k_\nu}{m^2}\right)}$$

k

Vector field for mass $m=0$ (photon)

$A_\mu(x)$ 4-potential. $\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

e.o.m. $\underbrace{\left(\Box g^{\mu\nu} - \partial^\mu \partial^\nu \right)}_{= K^{\mu\nu}} A_\nu = 0$ (Maxwell's eqs.)

- 2 physical solutions $\sim \varepsilon_\mu^{(\lambda)} e^{\pm ikx}$ with $\varepsilon \cdot k = 0$, $\vec{\varepsilon} \cdot \vec{k} = 0$
  transverse polarization
- unphysical solution: $\varepsilon_\mu \sim k_\mu$ longitudinal polarization
  $A_\mu(x) = k_\mu e^{\pm ikx} = \partial_\mu \left(\mp i\, e^{\pm ikx} \right) \equiv \partial_\mu \chi$
  $A_\mu \to A_\mu + \partial_\mu \chi$ gauge transformation

$K^{\mu\nu} k_\nu = 0$: k_ν eigenvector with eigenvalue $= 0$, $\det(K^{\mu\nu}) = 0$

$(K^{\mu\nu})^{-1}$ does not exist $\rightarrow$ propagator $K^{\mu\nu} D_{\nu\rho} = g^\mu_{\ \rho}$ (?)

- reason: gauge invariance of $\mathcal{L}$
- way out: break gauge invariance by $\mathcal{L} \to \mathcal{L} + \mathcal{L}_{fix}$

$\mathcal{L}_{fix} = -\frac{1}{2\xi} \left(\partial_\mu A^\mu\right)^2$ "gauge fixing term", ξ real parameter (free)

$K^{\mu\nu} \to K^{\mu\nu}$

$\Rightarrow$ propagator:

$$iD_{\mu\nu}(k) = \frac{i}{q^2 + i\varepsilon} \left[-g_{\mu\nu} + (1-\xi) \frac{k_\mu k_\nu}{k^2} \right]$$

$\xi = 1$: "Feynman gauge" $\sim \frac{-i g_{\mu\nu}}{q^2 + i\varepsilon}$

Remark: $\mathcal{L}_{fix}$ has no physical impact:

$k_\mu \leftrightarrow \partial_\mu$

photon couples to conserved current

$\partial_\mu j^\mu = 0$

1.5

Dirac field [spin $\frac{1}{2}$, mass m]

- spinor: $\psi(x) = \begin{pmatrix} \psi_1 \\ \vdots \\ \psi_4 \end{pmatrix}$, adjoint spinor: $\bar{\psi} = \psi^{+}\gamma^{0} = (\psi_1^*, \psi_2^*, -\psi_3^*, -\psi_4^*)$

- Dirac matrices: γ^μ $(\mu = 0,1,2,3)$, $\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $\gamma^k = \begin{pmatrix} 0 & \sigma_k \\ -\sigma_k & 0 \end{pmatrix}$

  notation: $\not{a} = a_\mu \gamma^\mu = \gamma^\mu a_\mu$ $\qquad$ σ_k: Pauli matrices

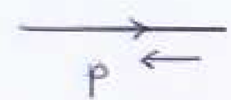
- Lagrangian: $\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi$

- Dirac eq. $\delta S = 0 \xrightarrow{\text{e.o.m.}}$ $\boxed{(i\gamma^\mu \partial_\mu - m)\psi = 0}$

- two types of solutions:

  ① $u(p)\, e^{-ipx}$: $(\not{p} - m)\, u(p) = 0$ $\qquad$ ② $v(p)\, e^{ipx}$: $(\not{p} + m)\, v(p) = 0$

particle



anti-particle

1.6

<u>propagator</u> : point-like source at $y \longrightarrow S(x-y)$

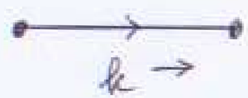
$$(i\gamma^\mu \partial_\mu - m)\, S(x-y) = \mathbb{1}\ \delta^4(x-y) \xrightarrow[\text{Transf.}]{\text{Fourier}} (\not k - m)\, S(k) = \mathbb{1}$$

solution (with $+i\varepsilon$ convention):

$$iS(k) = \frac{i}{\not k - m + i\varepsilon} = \frac{i(\not k + m)}{k^2 - m^2 + i\varepsilon}$$

$$S(x-y) = \int \frac{d^4k}{(2\pi)^4}\, S(k)\, e^{ik(x-y)}$$

causality behaviour



arrow = direction of charge flow of the <u>particle</u> (momentum space)

$S(x-y)$ describes

- particle propagation from $y \to x$ if $y^0 < x^0$
- anti-particle propagation from $x \to y$ if $x^0 < y^0$

Interaction higher powers of fields in $\mathcal{L}$ (> 2)

Example 1: Yukawa interaction

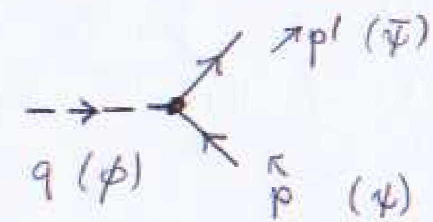
spinor $\psi(x)$, mass m + scalar $\phi(x)$, mass M

$$\mathcal{L} = \underbrace{\bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi + \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{\;}{2}\phi^2}_{\text{free part, } \mathcal{L}_0} + \underbrace{g\,\bar{\psi}\psi\phi}_{\mathcal{L}_{int}}$$

free part, $\mathcal{L}_0$

$\mathcal{L}_{int}$, g: coupling constant

graphical symbol for interaction: vertex



momentum conservation:

$$q = p' - p$$

1.8

Example 2: QED Quantum Electrodynamics

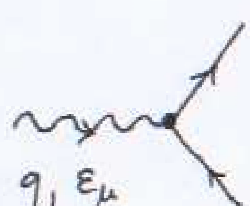
spinor $\psi(x)$, mass m + vector $A_\mu(x)$, mass $= 0$

$$\mathcal{L} = \underbrace{\bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \mathcal{L}_{fix}}_{\text{free part, } \mathcal{L}_0} + \underbrace{e\, \bar{\psi}\gamma^\mu \psi A_\mu}_{\mathcal{L}_{int} = j^\mu A_\mu}$$

$j^\mu = \bar{\psi}\gamma^\mu\psi$ current

e : coupling constant (charge)

vertex :



$q = p' - p$ momentum conservation

1.9

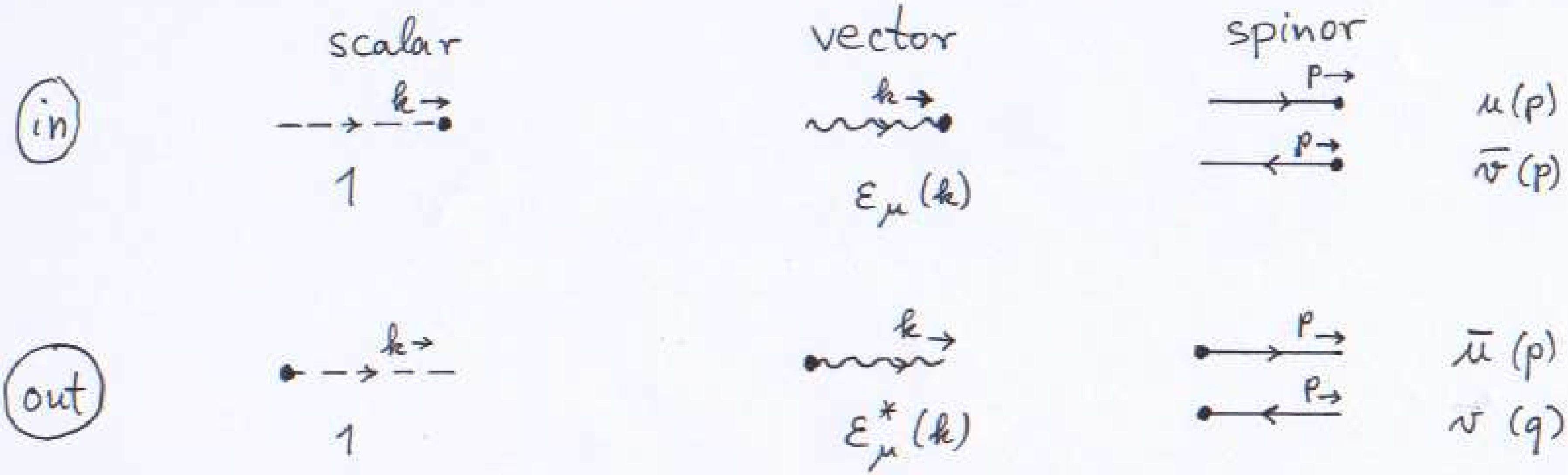
Feynman graphs

perturbation theory for scattering processes
= expansion in coupling constant(s)

QED: $\alpha = \frac{e^2}{4\pi} = \frac{1}{137}$ small $(\alpha > \alpha^2 > \alpha^3 \dots)$

- <u>Feynman rules</u>: (i) wave functions (ii) propagators (iii) vertices

(i) solution of field equations (e.o.m.) in momentum space



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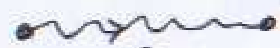
(ii) propagators from inhomogeneous wave eqs, point-like source

scalar

k

$$\frac{i}{k^2-m^2+i\varepsilon}$$

vector



k

$$\frac{i}{k^2-m^2+i\varepsilon}\left(-g_{\mu\nu}+\frac{k_\mu k_\nu}{m^2}\right)$$

$$\frac{i}{k^2+i\varepsilon}\cdot(-g_{\mu\nu})$$

for $m=0$

spinor

k

$$\frac{i(\not k+m)}{k^2-m^2+i\varepsilon}$$

(iii) vertices from $\mathcal{L}_{int}$ $\longrightarrow$ momentum space

Yukawa

$i g \mathbf{1}$

QED

$i e \gamma^\mu$

... (others)

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- scattering amplitudes $\equiv$ S-matrix elements

$$a_1 + a_2 \longrightarrow b_1 + b_2 + \cdots + b_n\,, \qquad a \longrightarrow b_1 + \cdots + b_n$$

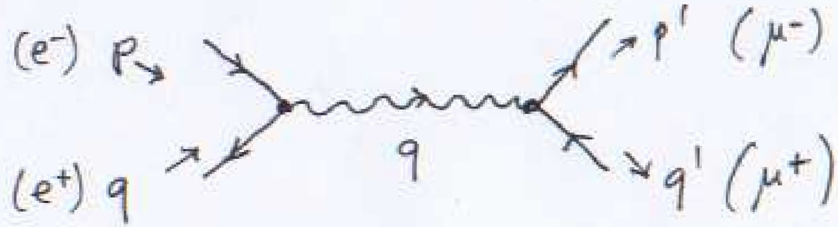
matrix element $\langle b_1 \cdots b_m | S | a_1 a_2 \rangle$, $\quad S = \lim_{\substack{t_0 \to -\infty \\ t \to \infty}} \mathcal{U}(t, t_0)$

$$\equiv \langle f | S | i \rangle \equiv S_{fi}$$

- Feynman graphs $\Rightarrow S_{fi}$ in given order of perturbation theory

<u>lowest order</u>: connect in- and outgoing particles by minimum number of vertices and propagators

<u>example (i)</u>: $e^+e^- \longrightarrow \mu^+\mu^-$ in QED

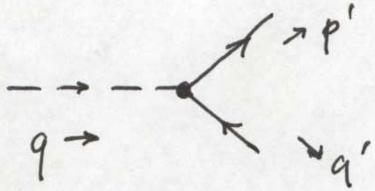


$[\,q = p + q = p' + q'\,]$

$$\bar{v}(q)\, ie\gamma^\mu u(p) \left(\frac{-i g_{\mu\nu}}{q^2 + i\varepsilon} \right) \bar{u}(p')\, ie\gamma^\nu v(q') \quad \sim \quad e^2 = \mathcal{O}(\alpha)$$

1.12

example (ii) : $\phi^0 \longrightarrow f\bar{f}$ (scalar ϕ^0 decays to $f\bar{f}$)



$q = p' + q'$, $q^2 = M^2$

M = mass of ϕ^0

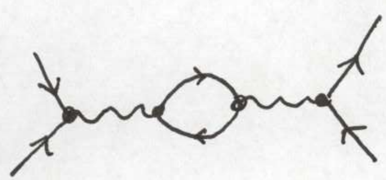
$\bar{u}(p')\, ig\, v(q') \sim g$

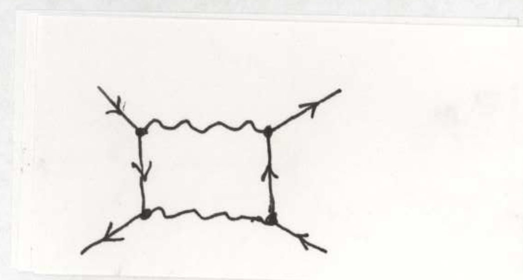
Higher Order

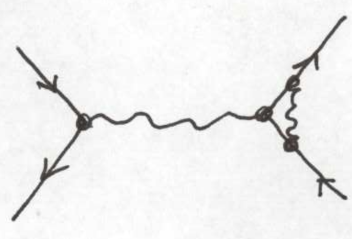
Feynman graphs with closed loops

example :

$e^+e^- \longrightarrow \mu^+\mu^-$







$\sim e^4$

2. Cross sections and decay widths

amplitudes

scattering process: $a + b \quad \longrightarrow \quad b_1 + b_2 + \cdots + b_n$

$$|a(p_a), b(p_b)> = |i>, \quad |b_1(p_1), \cdots b_n(p_n)> = |f>$$

matrix element = probability amplitude for $i \longrightarrow f$:

$$S_{fi} =< f|S|i >$$

amplitudes

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matrix element = probability amplitude for $i \rightarrow f$:

$$S_{fi} = < f|S|i >$$

for $i \neq f$: $\quad |S_{fi}|^2 = (2\pi)^4 \, \delta^4(P_i - P_f) \, |\mathcal{M}_{fi}|^2 \, (2\pi)^{-3(n+2)}$

$P_i = p_a + p_b \; = \; P_f = p_1 + \cdots + p_n$ momentum conservation

factors $(2\pi)^{-3}$ from wave function normalization (plane waves)

$\mathcal{M}_{fi}$ from Feynman graphs and rules probability for scattering into phase space element $\mathrm{d}\Phi$:

$$\mathrm{d}w_{fi} = |S_{fi}|^2 \, \mathrm{d}\Phi, \qquad \mathrm{d}\Phi = \frac{\mathrm{d}^3 p_1}{2p_1^0} \cdots \frac{\mathrm{d}^3 p_n}{2p_n^0}$$

$$\frac{\mathrm{d}^3 p_i}{2p_i^0} = \mathrm{d}^4 p_i \, \delta(p_i^2 - m_i^2)$$ Lorentz invariant phase space

differential cross section:

$$\mathrm{d}\sigma = \underbrace{\frac{(2\pi)^6}{4\sqrt{(p_a \cdot p_b)^2 - m_a^2 m_b^2}}} \; \mathrm{d}w_{fi}$$

flux normalization factor from initial state

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differential cross section:

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flux normalization factor from initial state

$$\boxed{\mathrm{d}\sigma = \frac{(2\pi)^4}{4\sqrt{(p_a \cdot p_b)^2 - m_a^2 m_b^2}} \; |\mathcal{M}_{fi}|^2 \, (2\pi)^{-3n} \, \delta^4(P_i - P_f) \, \frac{\mathrm{d}^3 p_1}{2p_1^0} \cdots \frac{\mathrm{d}^3 p_n}{2p_n^0}}$$

decay process: $a \quad \longrightarrow \quad b_1 + b_2 + \cdots + b_n$

$$|a(p_a)\!> = |i\!>, \quad |b_1(p_1), \cdots b_n(p_n)\!> = |f\!>$$

$$|S_{fi}|^2 = (2\pi)^4 \, \delta^4(p_a - P_f) \, |\mathcal{M}_{fi}|^2 \, (2\pi)^{-3(n+1)}$$

decay width (differential):

$$\mathrm{d}\Gamma = \underbrace{\frac{(2\pi)^3}{2\, m_a}}_{\textit{normalization of } |a>} \cdot \underbrace{|S_{fi}|^2 \, \mathrm{d}\Phi}_{\mathrm{d}w_{fi}}$$

decay process: $a \quad \longrightarrow \quad b_1 + b_2 + \cdots + b_n$

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$$\boxed{\mathrm{d}\Gamma = \frac{(2\pi)^4}{2\,m_a}\, |\mathcal{M}_{fi}|^2\, (2\pi)^{-3n}\, \delta^4(p_a - P_f)\, \frac{\mathrm{d}^3p_1}{2p_1^0} \cdots \frac{\mathrm{d}^3p_n}{2p_n^0}}$$

special case: 2-particle phase space

$$a + b \longrightarrow b_1 + b_2, \qquad a \longrightarrow b_1 + b_2$$

- **cross section**

  in the CMS, $\vec{p}_a + \vec{p}_b = 0 = \vec{p}_1 + \vec{p}_2$

$$\boxed{\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{1}{64\pi^2 s}\,\frac{|\vec{p}_1|}{|\vec{p}_a|}\,|\mathcal{M}_{fi}|^2}$$

$$\mathrm{d}\Omega = \mathrm{d}\cos\theta\,\mathrm{d}\phi, \qquad \theta = \sphericalangle(\vec{p}_a, \vec{p}_1)$$

$$s = (p_a + p_b)^2 = E^2_{\mathrm{CMS}}$$

- **decay rate**

  for final state masses $m_1 = m_2 = m$

$$\boxed{\frac{\mathrm{d}\Gamma}{\mathrm{d}\Omega} = \frac{1}{64\pi^2\,m_a}\sqrt{1 - \frac{4m^2}{m_a^2}}\;|\mathcal{M}_{fi}|^2}$$

3. Gauge theories

3.1 Abelian gauge theories – QED

free fermion field ψ (for $e^{\pm}$), described by Lagrangian

$$\mathcal{L}_0 = \overline{\psi}\,(i\gamma^\mu\partial_\mu - m)\,\psi$$

- $\mathcal{L}_0$ is invariant under global transformations

  $\psi(x) \;\to\; \psi'(x) = e^{i\alpha}\,\psi(x)$ *with α real, arbitrary*

  group: $\boldsymbol{U(1)}$, global $\boldsymbol{U(1)}$

  global gauge symmetry

- $\mathcal{L}_0$ is not invariant under local transformations

  $$\psi(x) \;\to\; \psi'(x) = \underbrace{e^{i\alpha(x)}}_{U(x)}\psi(x)$$

invariance is obtained by “minimal substitution”

$$\partial_\mu \;\rightarrow\; D_\mu = \partial_\mu - ieA_\mu \qquad \textit{covariant derivative}$$

under the combined transformations

$$\boxed{\begin{aligned} \psi(x) \;\rightarrow\; \psi'(x) &= e^{i\alpha(x)}\,\psi(x) \equiv U(x)\,\psi(x) \\ A_\mu(x) \;\rightarrow\; A'_\mu(x) &= A_\mu(x) + \frac{1}{e}\,\partial_\mu\alpha(x) \end{aligned}}$$

local gauge transformations

group: local $\boldsymbol{U(1)}$, Abelian: $e^{i\alpha_1}e^{i\alpha_2} = e^{i\alpha_2}e^{i\alpha_1}$

basic property: $\boxed{D'_\mu\psi' = U(x)D_\mu\psi}$

$$\underbrace{(\partial_\mu - ieA'_\mu)}_{D'_\mu}\,\underbrace{U(x)\psi(x)}_{\psi'} = U(x)\,\underbrace{(\partial_\mu - ieA_\mu)\,\psi}_{D_\mu\psi}$$

$\Rightarrow \mathcal{L}$ is invariant under local gauge transformations:

$$\boxed{\mathcal{L}' = \overline{\psi'}\,(i\gamma^\mu D'_\mu - m)\,\psi' \;=\; \overline{\psi}\,(i\gamma^\mu D_\mu - m)\,\psi = \mathcal{L}}$$

proof with $\quad \overline{\psi'} = \overline{U\psi} = \overline{\psi}U^* = \overline{\psi}U^{-1} \quad$ *and* $\quad D'_\mu \psi' = U D_\mu \psi$

The invariant Lagrangian contains a new vector field A_μ which couples to the electromagnetic current:

$$\mathcal{L} = \overline{\psi}\,(i\gamma^\mu D_\mu - m)\,\psi = \mathcal{L}_0 \,+\, e\,\overline{\psi}\gamma^\mu\psi\,A_\mu = \mathcal{L}_0 + \mathcal{L}_{int}$$

local gauge invariance determines the interaction

A_μ not yet a dynamical field $\quad \Rightarrow \quad$ add $\mathcal{L}_A$ (invariant!)

$\mathcal{L}_A = -\frac{1}{4}\,F_{\mu\nu}F^{\mu\nu}\,(+\mathcal{L}_{fix})$ *free photon field*

$$\boxed{\mathcal{L}_{\rm QED} = \mathcal{L}_0 + \mathcal{L}_A + \mathcal{L}_{int}}$$

QED as a gauge theory: main steps

- start with $\mathcal{L}_0(\psi, \partial_\mu \psi))$ for free fermion field ψ
  symmetric under global gauge transformations

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- perform minimal substitution $\rightarrow \mathcal{L}_0(\psi, D_\mu \psi)$

$$\partial_\mu \;\rightarrow\; D_\mu = \partial_\mu - ieA_\mu$$

$\Rightarrow$ invariance under local gauge transformations

  - involves additional vector field A_μ
  - induces interaction between A_μ and ψ

$$e\, \psi\gamma^\mu\psi\; A_\mu \;\equiv\; e\; j^\mu A_\mu$$

QED as a gauge theory: main steps

- start with $\mathcal{L}_0(\psi, \partial_\mu\psi))$ for free fermion field ψ symmetric under global gauge transformations
- perform minimal substitution $\rightarrow \mathcal{L}_0(\psi, D_\mu\psi)$

$$\partial_\mu \;\rightarrow\; D_\mu = \partial_\mu - ieA_\mu$$

$\Rightarrow$ invariance under local gauge transformations

  - involves additional vector field A_μ
  - induces interaction between A_μ and ψ

$$e\,\psi\gamma^\mu\psi\;A_\mu \;\equiv\; e\;j^\mu A_\mu$$

- make A_μ a dynamical field by adding

$$\mathcal{L}_A = -\tfrac{1}{4}\,F_{\mu\nu}\,F^{\mu\nu}\;(+\mathcal{L}_{fix})$$

3.2 Non-Abelian gauge theories

Generalization: “phase” transformations that do not commute

$$\psi \to \psi' = U\psi \quad \textit{with} \quad U_1 U_2 \neq U_1 U_1$$

requires matrices, *i.e.* ψ is a multiplet

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \vdots \\ \psi_n \end{pmatrix}, \qquad U = n \times n \textit{ -matrix}$$

each $\psi_k = \psi_k(x)$ is a Dirac spinor

(i) global symmetry

starting point: $\mathcal{L}_0 = \overline{\psi}\, i\gamma^\mu\, \partial_\mu \psi$

where $\overline{\psi} = (\overline{\psi}_1, \cdots, \overline{\psi}_n)$

consider unitary matrices: $U^\dagger = U^{-1}$

$$\psi' = U\psi \quad \Rightarrow \quad \overline{\psi'} = \overline{\psi}\, U^\dagger = \overline{\psi}\, U^{-1}$$

$$\Rightarrow \quad \overline{\psi'}\psi' = \overline{\psi}\psi, \quad \overline{\psi'}\gamma^\mu\partial_\mu\psi' = \overline{\psi}\gamma^\mu\partial_\mu\psi$$

if U does not depend on x

$\Rightarrow$ $\mathcal{L}_0$ is invariant under $\psi \to U\psi$

U: global gauge transformation

similar for **scalar fields:**

$$\phi \longrightarrow \psi' = U\phi, \qquad \phi = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_n \end{pmatrix}$$

each $\phi_k = \phi_k(x)$ is a scalar field, $\phi^\dagger = (\phi_1^\dagger, \cdots, \phi_n^\dagger)$

terms $\phi^\dagger\phi$, $(\partial_\mu\phi)^\dagger(\partial^\mu\phi)$ are invariant

$\Rightarrow \quad \mathcal{L}_0 = (\partial_\mu\phi)^\dagger(\partial^\mu\phi)$ is invariant relevant in physics:

the special unitary $n \times n$-matrices with det=1

group $SU(n)$

<u>examples:</u>

$$SU(2): \quad \psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \quad e.g. \quad \psi = \begin{pmatrix} \psi_\nu \\ \psi_e \end{pmatrix}$$ *isospin*

$$SU(3): \quad \psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} \quad e.g. \quad \psi = \begin{pmatrix} \psi_r \\ \psi_g \\ \psi_b \end{pmatrix}$$ *colour*

$SU(n)$ matrices U can be written as exponentials

$$U(\theta_1, \cdots, \theta_N) = e^{i\theta_a T_a} \qquad \textit{sum over } a = 1, \cdots n$$

$\theta_1, \cdots \theta_N$: real parameters

$T_1, \cdots T_N$: $n \times n$-matrices, generators, $T_a^\dagger = T_a$

infinitesimal θ : $U = \mathbf{1} + i\theta_a T_a \quad (+O(\theta^2))$

N-dimensional Lie Group

det=1 and unitarity $\Rightarrow$ $N = n^2 - 1$

$n = 2$: $N = 3$, $n = 3$: $N = 8$

commutators $[T_a, T_b] \neq 0$ non-Abelian

$$\boxed{[T_a, T_b] = i\, f_{abc}\, T_c}$$

Lie Algebra

f_{abc} : real numbers, structure constants

$f_{abc} = -f_{bac} = \cdots$ antisymmetric

commutators $[T_a, T_b] \neq 0$ non-Abelian

$$\boxed{[T_a, T_b] = i\, f_{abc}\, T_c}$$

Lie Algebra

f_{abc} : real numbers, structure constants

$f_{abc} = -f_{bac} = \cdots$ antisymmetric

$\boxed{SU(2)}$ $f_{abc} = \epsilon_{abc}$ *(like angular momentum)*

$T_a = \frac{1}{2}\,\sigma_a,$ σ_a : *Pauli matrices (a=1,2,3)*

commutators $[T_a, T_b] \neq 0$ non-Abelian

$$\boxed{[T_a, T_b] \;=\; f_{abc}\, T_c}$$

Lie Algebra

f_{abc} : real numbers, structure constants

$f_{abc} = -f_{bac} = \cdots$ antisymmetric

$\boxed{SU(2)}$ $f_{abc} = \epsilon_{abc}$ *(like angular momentum)*

$T_a = \frac{1}{2}\,\sigma_a,$ σ_a : *Pauli matrices (a=1,2,3)*

$\boxed{SU(3)}$ $T_a = \frac{1}{2}\,\lambda_a,$ λ_a : *Gell-Mann matrices (a=1,...8)*

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \ldots$$

(ii) local symmetry

now: $\Theta_a = \Theta_a(x)$, $a = 1, \dots N$ real functions

$$\psi(x) \to \mathcal{U}\big(\Theta_1(x), \dots \Theta_N(x)\big)\,\psi(x) \equiv \mathcal{U}(x)\,\psi(x) = \psi'(x)$$

$$\psi'(x) = \mathcal{U}(x)\,\psi(x) \Longrightarrow \overline{\psi'(x)} = \bar{\psi}(x)\,\mathcal{U}^{-1}(x)$$

but: $\partial_\mu \psi'(x) = \partial_\mu\big(\mathcal{U}(x)\psi(x)\big) \neq \mathcal{U}(x)\,\partial_\mu \psi(x)$

no symmetry of $\mathcal{L}_0 = \bar{\psi}\, i\gamma^\mu \partial_\mu \psi$

strategy : invent covariant derivative $\partial_\mu \to D_\mu$

require
$$\boxed{D'_\mu\, \psi'(x) = \mathcal{U}(x)\, D_\mu\, \psi(x)}$$

$$\Longrightarrow \overline{\psi'}\, i\gamma^\mu D'_\mu \psi' = \bar{\psi}\, \mathcal{U}^{-1}\, i\gamma^\mu\, \mathcal{U}\, D_\mu \psi = \bar{\psi}\, i\gamma^\mu D_\mu \psi \quad \text{inv.}$$

same for scalar fields:

$$\partial_\mu \phi \longrightarrow D_\mu \phi$$

$$D'_\mu \phi' = U(x)\, D_\mu \phi \quad \Rightarrow \quad (D'_\mu \phi')^+ = (D_\mu \phi)^+ \, U^{-1}(x)$$

$$\Rightarrow \quad (D'_\mu \phi')^+ (D'^\mu \phi') = (D_\mu \phi)^+ (D^\mu \phi) \quad = \mathcal{L}_0 \quad \text{invariant}$$

- how does D_μ look like?

must involve vector field, must be a matrix

$$D_\mu = \partial_\mu - i g \mathcal{W}_\mu(x)$$

g: constant

$\mathcal{W}_\mu$: $n \times n$ - matrix

expand in terms of generators T_a:

$$\mathcal{W}_\mu(x) = T_a W_\mu^a(x)$$

[sum over $a = 1, \dots N$]

contains N vector fields $W_\mu^a(x)$: gauge fields

SU(2): $W_\mu^1, W_\mu^2, W_\mu^3$

SU(3): $W_\mu^1, \dots W_\mu^8$

- how does $W_\mu(x)$ resp. D_μ transform?

condition: $D'_\mu \psi' = \mathcal{U}(x) D_\mu \psi$ with $D'_\mu = \partial_\mu - ig W'_\mu$

$$(\partial_\mu - ig W'_\mu)\,\mathcal{U}\psi = \mathcal{U}\,(\partial_\mu - ig W_\mu)\,\psi \qquad \text{for all } \psi$$

fullfilled if

$$\boxed{W'_\mu = \mathcal{U} W_\mu \mathcal{U}^{-1} - \frac{i}{g}(\partial_\mu \mathcal{U})\,\mathcal{U}^{-1}}$$

* $\psi \to \psi' = \mathcal{U}(x)$ and $W_\mu \to W'_\mu$: local gauge transformation

for infinitesimal Θ (neglect $\theta^2, \ldots$ terms) : $\mathcal{U} = 1 + i\theta_a T_a$

$$* \quad W'^{\,a}_\mu = \underbrace{W^a_\mu + \frac{1}{g}\partial_\mu \Theta_a}_{\text{as Abelian}} + \underbrace{f_{abc}\, W^b_\mu\, \Theta_c}_{\text{new, non-Abelian}}$$

- substitution $\partial_\mu \to D_\mu$ in $\mathcal{L}_0$ induces interactions

Spin $\frac{1}{2}$

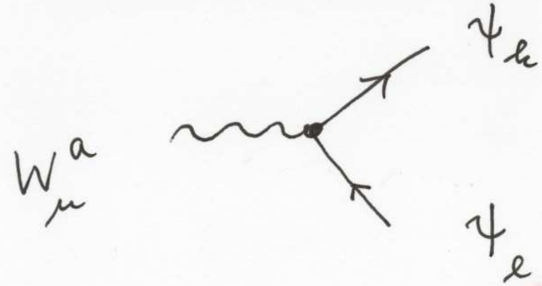
$$\mathcal{L}_0 \to \mathcal{L}_0 + \underbrace{g\,\bar{\psi}\gamma^\mu W_\mu \psi}_{} = \mathcal{L}_0 + \mathcal{L}_{int}$$

$$= g\left(\bar{\psi}\gamma^\mu T_a \psi\right)\cdot W_\mu^a = g\, j_a^\mu\, W_\mu^a$$

N currents j_a^μ

$$j_a^\mu = \bar{\psi}_k \gamma^\mu (T_a)_{kl} \psi_l \equiv (\bar{\psi}_1, \ldots \bar{\psi}_N)\gamma^\mu T_a \begin{pmatrix}\psi_1 \\ \vdots \\ \psi_N\end{pmatrix}$$

vertices:



$$i g\, (T_a)_{kl}\, \gamma^\mu$$

Spin 0

$$\mathcal{L}_0 \rightarrow (D_\mu \phi)^+ (D^\mu \phi)$$

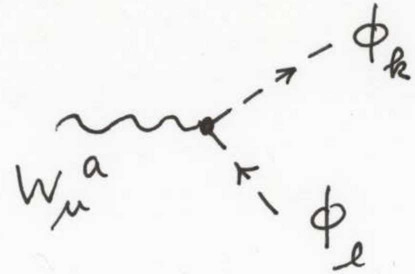
$$= (\partial_\mu \phi - ig \mathcal{W}_\mu \phi)^+ \, (\partial^\mu \phi - ig \mathcal{W}^\mu \phi)$$

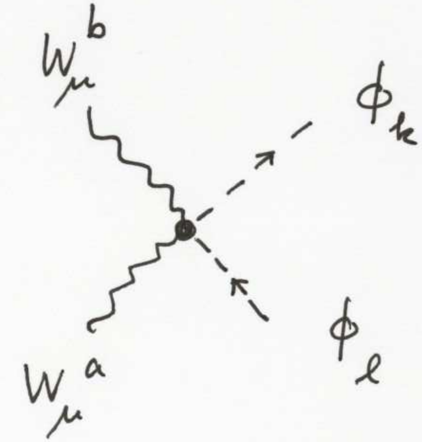
$$= (\partial_\mu \phi)^+ (\partial^\mu \phi) + g\left(\phi^+ \mathcal{W}_\mu \cdot i\partial^\mu \phi - i\partial_\mu \phi^+ \cdot \mathcal{W}_\mu \phi\right)$$

$$+ g^2 \phi^+ \mathcal{W}_\mu \mathcal{W}^\mu \phi$$

$$= \mathcal{L}_0 + \mathcal{L}_{int}$$

vertices:





[note: $i\partial_\mu \rightarrow p_\mu$ in momentum space $\rightarrow$ Feynman rules]

What is dynamics of W^a_μ fields?

need: additional term $\mathcal{L}_W \to$ e.o.m., propagators

naive: $\sum_a (\partial_\mu W^a_\nu - \partial_\nu W^a_\mu)^2$ ↯ not gauge invariant

try: $\mathcal{F}_{\mu\nu} = \mathcal{D}_\mu \mathcal{W}_\nu - \mathcal{D}_\nu \mathcal{W}_\mu \equiv F^a_{\mu\nu} T_a$

$$= \partial_\mu \mathcal{W}_\nu - \partial_\nu \mathcal{W}_\mu - ig [\mathcal{W}_\mu, \mathcal{W}_\nu]$$

$$= \frac{i}{g} [\mathcal{D}_\mu, \mathcal{D}_\nu]$$

gauge transformation: $\mathcal{W}_\mu \to \mathcal{W}'_\mu$, $\mathcal{D}_\mu \to \mathcal{D}'_\mu$

$$\Rightarrow \mathcal{F}_{\mu\nu} \to \mathcal{F}_{\mu\nu}' = \mathcal{U} \mathcal{F}_{\mu\nu} \mathcal{U}^{-1}$$

$$\Rightarrow \mathrm{Tr}(\mathcal{F}'_{\mu\nu} \mathcal{F}'^{\mu\nu}) = \mathrm{Tr}(\mathcal{U} \mathcal{F}_{\mu\nu} \mathcal{U}^{-1} \mathcal{U} \mathcal{F}^{\mu\nu} \mathcal{U}^{-1}) = \mathrm{Tr}(\mathcal{F}_{\mu\nu} \mathcal{F}^{\mu\nu})$$

gauge invariant

Lagrangian:

$$\mathcal{L}_W = -\frac{1}{2} \mathrm{Tr}\left(\mathcal{F}_{\mu\nu} \mathcal{F}^{\mu\nu}\right)$$
$$= -\frac{1}{4} \sum_a F_{\mu\nu}{}^a F^{a,\mu\nu}$$

components of $\mathcal{F}_{\mu\nu}$:

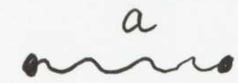
$$F_{\mu\nu}{}^a = \partial_\mu W_\nu{}^a - \partial_\nu W_\mu{}^a + g f_{abc} W_\mu{}^b W_\nu{}^c$$

[makes use of normalization $\mathrm{Tr}(T_a T_b) = \frac{1}{2}\delta_{ab}$]

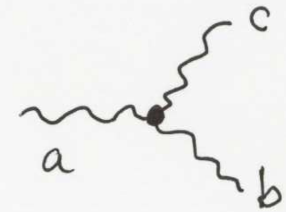
$$F_{\mu\nu}{}^a = \text{"Abelian"} + \text{non-Abelian}$$

$$\mathcal{L}_W = \underbrace{(\text{quadratic})}_{\text{free part}} + \underbrace{(\text{cubic}) + (\text{quartic})}_{\text{tri- and quadri-linear interactions}}$$

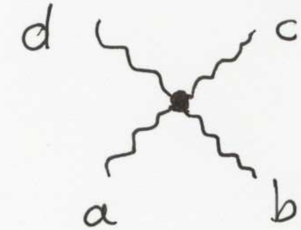
$$\mathcal{L}_W = -\frac{1}{4} \left(\partial_\mu W_\nu^a - \partial_\nu W_\mu^a\right)^2$$



$$-\frac{1}{2} g f_{abc} \left(\partial_\mu W_\nu^a - \partial_\nu W_\mu^a\right) W^{b,\mu} W^{c,\nu}$$



$$-\frac{1}{4} g^2 f_{abc} f_{ade} W_\mu^b W_\nu^c W^{d,\mu} W^{e,\nu}$$



new type of couplings:

self-couplings of vector fields

- gauge couplings

g: universal coupling constant

3.3. Quantum Chromodynamics (QCD)

each quark field $q = u, d, \ldots$ appears in 3 colours

$$\psi = \begin{pmatrix} q \\ q \\ q \end{pmatrix}, \qquad \overline{\psi} = (\overline{q}, \overline{q}, \overline{q})$$

$T_a = \frac{1}{2}\,\lambda_a \quad (a = 1, \ldots 8)$ *generators*

$W_\mu^a \equiv G_\mu^a$ *8 gluon fields*

$G_{\mu\nu}^a = \partial_\mu G_\nu^a - \partial_\nu G_\mu^a + g_s\, f_{abc}\, G_\mu^b G_\nu^c$ *field strength*

$D_\mu = \partial_\mu - i g_s\, T_a\, G_\mu^a$ *covariant derivative*

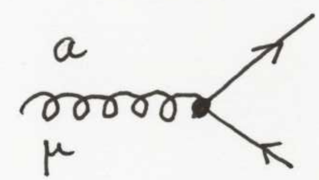
g_s *coupling constant of strong interaction,* $\alpha_s = \frac{g_s^2}{4\pi}$

QCD Lagrangian

$$\mathcal{L}_{QCD} = \bar{\psi}\,(i\gamma^\mu D_\mu - m)\,\psi + \mathcal{L}_G =$$

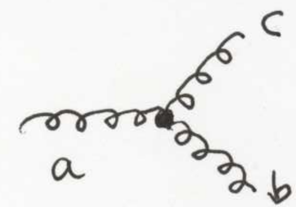
$q_k \; (k = 1,2,3)$

$$\bar{\psi}(i\gamma^\mu - m)\,\psi$$

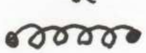


$$+\; g_s\, \bar{\psi}\, \gamma^\mu \frac{\lambda_a}{2}\, \psi \; G^a_\mu$$

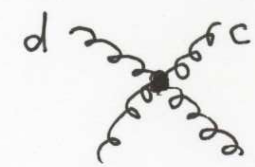
$$-\; \frac{1}{4}\, G^a_{\mu\nu}\, G^{a,\mu\nu} \;-\; \underbrace{\frac{1}{2\xi}\left(\partial_\mu G^{a,\mu}\right)^2}_{\mathcal{L}_{fix}}$$

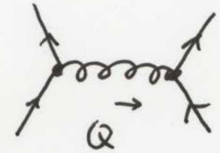


for $\xi = 1$:



$$-\frac{i\, g_{\mu\nu}}{k^2 + i\varepsilon}$$

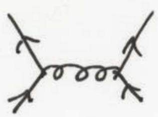
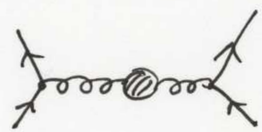


quark-quark interaction:  $\sim \frac{g_s^2}{Q^2} = \frac{4\pi\alpha_s}{Q^2}$

higher order:  +  + ...

self-energy $\quad \Pi(Q^2) = \frac{\alpha_s}{4\pi}\left(\frac{2}{3}n_f - 11\right)\log Q^2 + \underbrace{\ldots}_{\text{independ. of } Q^2}$

$n_f = 5\,(6)$ number of flavours

summation:  +  +  + ...

$$\frac{\alpha_s}{Q^2}\left[1 + \Pi + \Pi^2 + \ldots\right] = \frac{1}{Q^2}\underbrace{\frac{\alpha_s}{1-\Pi(Q^2)}}_{\alpha_s(Q^2)}$$

running coupling

α_s: formal parameter of $\mathcal{L}$, $\alpha_s(Q_0^2)$ measurable at Q_0^2

$\Rightarrow$ eliminate α_s by $\alpha_s(Q_0^2)$, exp. input $[\alpha_s(M_Z^2) = 0.12]$

$$\Rightarrow \quad \alpha_s(Q^2) = \frac{\alpha_s(Q_0^2)}{1 + \frac{\alpha_s(Q_0^2)}{4\pi}\underbrace{\left(11 - \frac{2}{3}n_f\right)}_{=:\,\beta_0\,>\,0 \text{ for } n_f < \frac{33}{2}} \log\frac{Q^2}{Q_0^2}} \quad \rightarrow 0 \text{ for high } Q^2$$

fulfills diff. eq.

$$Q^2 \frac{\partial \alpha_s}{\partial Q^2} = -\frac{\beta_0}{4\pi}\alpha_s^2 \qquad \text{RGE}$$

$$= \beta(\alpha_s) \qquad \beta\text{-function}$$

in perturbation theory:

$$\beta(\alpha_s) = \underbrace{-\frac{\beta_0}{4\pi}\alpha_s^2}_{\text{1-loop}} \underbrace{-\frac{\beta_1}{(4\pi)^2}\alpha_s^3}_{\text{2-loop}} \ldots$$

[QED: $\beta_0 = -\frac{4}{3}\sum_f Q_f^2 < 0$]

4. Higgs mechanism

problem: weak interaction, gauge bosons are massive

mass terms $\sim M^2 W^a_\mu W^{a,\mu}$ spoil local gauge invariance

- bad high energy behaviour of amplitudes and cross sections, conflict with unitarity
  reason: longitudinal polarization $\epsilon^\mu \simeq \frac{k^\mu}{M} \sim k^\mu$
- bad divergence of higher orders with loop diagrams
  reason: propagators contain $-g_{\mu\nu} + \frac{k_\mu k_\nu}{M^2}$
  $\Rightarrow$ additional powers of momenta in loop integration
  $\Rightarrow$ spoil renormalizability

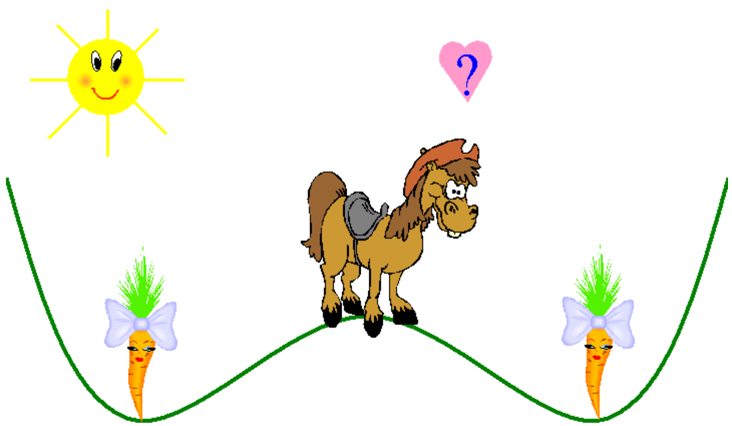
renormalizable theory:
divergences can be absorbed into the parameters

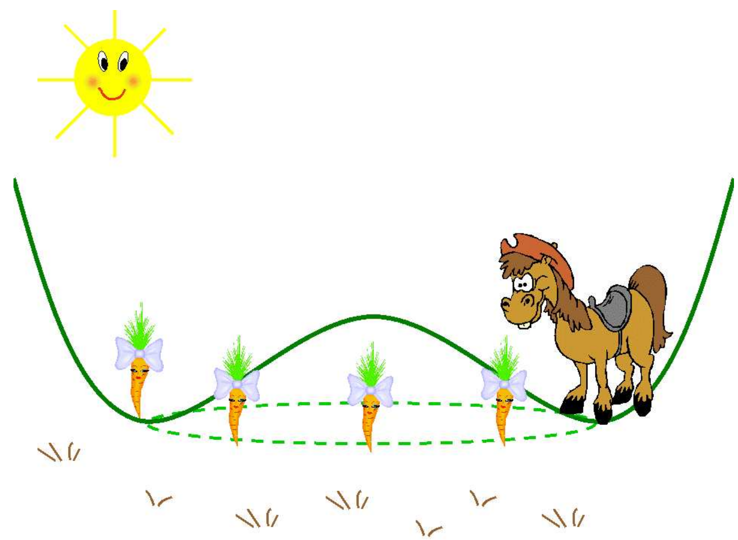
gauge invariant theories are renormalizable

4.1 Spontaneous symmetry breaking (SSB)

physical system: has a symmetry

ground state: not symmetric

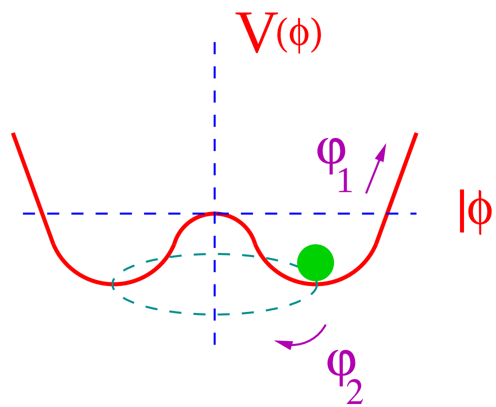




consider complex scalar field $\phi \neq \phi^\dagger$

Lagrangian with interaction V (potential), minimum at $\phi_0 = v$

$$\mathcal{L} = |\partial_\mu \phi|^2 \, - \, V(\phi)$$



$V = V(|\phi|)$: *$\mathcal{L}$ symmetric under* $\phi \to e^{i\alpha}\,\phi, \quad U(1)$

$v \neq 0$: $\phi_0' = e^{i\alpha}\, v \neq \phi_0$ *not symmetric*

$V = V(|\phi_0'|) = V(|\phi_0|)$: *vacuum is degenerate* write $\phi(x) = \eta(x) e^{i\Theta(x)}$, η and Θ real

$V(|\phi|) = V(\eta)$, minimum at $\eta = v$: $V'(v) = 0$, $V''(v) > 0$

expand around minimum: $\eta(x) = v + \frac{1}{\sqrt{2}} H(x)$

$$V(\eta) = \underbrace{V(v)}_{\text{const, drop}} + \frac{1}{2} V''(v) \cdot \frac{1}{2} H^2 + \cdots$$

$$\mathcal{L} = \frac{1}{2} (\partial_\mu H)(\partial^\mu H) - \underbrace{\left[\frac{1}{2} V''(v)\right]}_{= m_H^2 > 0} \cdot \frac{1}{2} H^2 + v^2 (\partial_\mu \Theta)(\partial^\mu \Theta) + \cdots$$

mass of H

H-field is massive

Θ-field is massless, (no Θ^2 term) "Goldstone field"

(special case of Goldstone Theorem

Goldstone Theorem (for non-Abelian case)

$$\phi = \begin{pmatrix} \phi_1 \\ \vdots \\ \phi_n \end{pmatrix}, \quad \mathcal{L} \text{ symmetric under } \phi \to \phi' = e^{i\theta_a T_a} \phi$$

$$\text{minimum at } \quad \phi_0 = \begin{pmatrix} \phi_1^0 \\ \vdots \\ \phi_n^0 \end{pmatrix}$$

$T_a \phi_0 \neq 0$ for $a = 1, \ldots K$ (spont. broken generators)

$T_a \phi_0 = 0$ for $a = K+1, \ldots N$ (unbroken gen.)

$\Longrightarrow$ there are K massless Goldstone fields

in QFT: fields describe Goldstone bosons (spin 0) with mass = 0

4.2. SSB in gauge theories

again scalar field $\phi \neq \phi^+$, $\mathcal{L}_0$ symmetric under $U(1)$

$\mathcal{L}_0 = (\partial_\mu \phi)^+ (\partial^\mu \phi) - V(|\phi|)$ $\longrightarrow$ symmetric under local $U(1)$

$$\mathcal{L} = (D_\mu \phi)^+ (D^\mu \phi) - V(|\phi|) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}, \qquad D_\mu = \partial_\mu - ie A_\mu$$

$$\left[\text{as before:} \quad \phi(x) = \eta(x) e^{i\theta(x)}, \qquad \eta(x) = v + \frac{1}{\sqrt{2}} H(x) \right]$$

$\mathcal{L}$ symmetric under

$$\begin{cases} \phi(x) \to \phi'(x) = e^{i\alpha(x)} \phi(x) = e^{i\alpha(x)} e^{i\theta(x)} \eta(x) \\ A_\mu(x) \to A'_\mu(x) = A_\mu(x) + \frac{1}{e} \partial_\mu \alpha(x) \end{cases}$$

choose $\alpha(x) = -\theta(x)$: $\quad \phi'(x) = \eta(x)$

$$\mathcal{L}(\phi', A'_\mu) = |(\partial_\mu - ie A'_\mu) \eta|^2 - V(\eta) - \frac{1}{4} F'_{\mu\nu} F'^{\mu\nu}$$

the massless θ-field has dissappeared! (unphysical field)

$$\mathcal{L} = \left| \left(\partial_\mu - ieA'_\mu\right)\left(v + \frac{H}{\sqrt{2}}\right)\right|^2 - \frac{1}{4} F'_{\mu\nu} F^{\mu\nu\prime} - V(\eta)$$

$$= \underbrace{-\frac{1}{4} F'_{\mu\nu} F'^{\mu\nu} + v^2 e^2 A'_\mu A'^\mu}_{\text{massive A-field, } M_A \sim v \cdot e} + \underbrace{\frac{1}{2}\left[(\partial_\mu H)^2 - m_H^2 H^2\right]}_{\text{neutral scalar, mass } m_H \neq 0} + \underbrace{\cdots\cdots}_{\text{int. terms}}$$

in this special gauge: no Goldstone boson **unitary gauge**

A_μ propagator:  k

$$\frac{i}{k^2 - M_A^2 + i\varepsilon} \underbrace{\left(-g_{\mu\nu} + \frac{k_\mu k_\nu}{M_A^2}\right)}_{\text{polarization sum of 3 polarization states}}$$

massive vector field without spoiling gauge symmetry of $\mathcal{L}$

two different gauges

properties	ϕ field	A_μ field
symmetry manifest	$H,\ \theta$	2 polarizations (transverse)
physics manifest	H	3 polarizations (2 transverse + 1 longitudinal)

$\theta \quad \longrightarrow \quad$ *longitudinal polarization of* A_μ

5. Electroweak interaction and Standard Model

preliminaries

<u>Dirac matrices:</u> $\gamma^\mu \; (\mu = 0, 1, 2, 3), \quad \gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2\, g^{\mu\nu}$

$\overline{\Gamma} = \gamma^0 (\Gamma)^\dagger \, \gamma^0, \quad \Gamma$ *any Dirac matrix oder product of matrices*

further Dirac matrix: $\gamma_5 = \gamma^0 \gamma^1 \gamma^2 \gamma^3 = \begin{pmatrix} 0 & \mathbf{1} \\ \mathbf{1} & 0 \end{pmatrix}$

$\gamma_5 \gamma^\mu + \gamma^\mu \gamma_5 = 0, \qquad \overline{\gamma_5} = -\gamma_5, \qquad \gamma_5^2 = \mathbf{1}$

<u>chiral fermions:</u>

$\psi^L = \frac{1-\gamma_5}{2}\, \psi$ *left-handed spinor, L-chiral spinor*

$\psi^R = \frac{1+\gamma_5}{2}\, \psi$ *right-handed spinor, R-chiral spinor*

projectors on left/right chirality: $\omega_\pm = \frac{1 \pm \gamma_5}{2}, \quad (\omega_\pm)^2 = \omega_\pm$

chiral currents:

$$\overline{\psi^L}\,\gamma^\mu \psi^L = \overline{\psi}\,\gamma^\mu \tfrac{1-\gamma_5}{2}\,\psi \equiv J_L^\mu$$ left-handed current

$$\overline{\psi^R}\,\gamma^\mu \psi^R = \overline{\psi}\,\gamma^\mu \tfrac{1+\gamma_5}{2}\,\psi \equiv J_R^\mu$$ right-handed current

$$J_V^\mu \;=\; \overline{\psi}\gamma^\mu\psi \;=\; J_L^\mu + J_R^\mu$$ vector current

$$J_A^\mu \;=\; \overline{\psi}\gamma^\mu\gamma_5\psi \;=\; -J_L^\mu + J_R^\mu$$ axialvector current

mass term:

$$m\,\overline{\psi}\psi \;=\; m\,(\overline{\psi^L}\,\psi^R \,+\, \overline{\psi^R}\,\psi^L)$$

connects L and R !

symmetry group: $SU(2)_I \times U(1)_Y$

$SU(2)_I$: weak isospin, generators $T_{\mathrm{I}}^a = \frac{1}{2}\,\sigma^a$ *for* L, $= 0$ *for* R

$U(1)_Y$: weak hypercharge, generator Y

$$T_{\mathrm{I}}^3 + Y/2 = Q$$

Fermion content of the SM:
(ignoring possible right-handed neutrinos)

					T_{I}^3	Q
leptons:	$\Psi_L^{\mathrm{L}} =$	$\begin{pmatrix} \nu_{\mathrm{e}}^{\mathrm{L}} \\ \mathrm{e}^{\mathrm{L}} \end{pmatrix},$	$\begin{pmatrix} \nu_{\mu}^{\mathrm{L}} \\ \mu^{\mathrm{L}} \end{pmatrix},$	$\begin{pmatrix} \nu_{\tau}^{\mathrm{L}} \\ \tau^{\mathrm{L}} \end{pmatrix},$	$+\frac{1}{2}$ $-\frac{1}{2}$	0 -1
	$\psi_l^{\mathrm{R}} =$	$\mathrm{e}^{\mathrm{R}},$	$\mu^{\mathrm{R}},$	$\tau^{\mathrm{R}},$	0	-1
quarks: (Each quark exists in 3 colours!)	$\Psi_Q^{\mathrm{L}} =$	$\begin{pmatrix} \mathrm{u}^{\mathrm{L}} \\ \mathrm{d}^{\mathrm{L}} \end{pmatrix},$	$\begin{pmatrix} \mathrm{c}^{\mathrm{L}} \\ \mathrm{s}^{\mathrm{L}} \end{pmatrix},$	$\begin{pmatrix} \mathrm{t}^{\mathrm{L}} \\ \mathrm{b}^{\mathrm{L}} \end{pmatrix},$	$+\frac{1}{2}$ $-\frac{1}{2}$	$+\frac{2}{3}$ $-\frac{1}{3}$
	$\psi_u^{\mathrm{R}} =$	$\mathrm{u}^{\mathrm{R}},$	$\mathrm{c}^{\mathrm{R}},$	$\mathrm{t}^{\mathrm{R}},$	0	$+\frac{2}{3}$
	$\psi_d^{\mathrm{R}} =$	$\mathrm{d}^{\mathrm{R}},$	$\mathrm{s}^{\mathrm{R}},$	$\mathrm{b}^{\mathrm{R}},$	0	$-\frac{1}{3}$

gauge boson content

$SU(2)_I$: generators $T_{\mathrm{I}}^1,\ T_{\mathrm{I}}^2,\ T_{\mathrm{I}}^3$

gauge fields $W_\mu^1,\ W_\mu^2,\ W_\mu^3$

also: $W_\mu^\pm = \frac{1}{\sqrt{2}}\left(W_\mu^1 \pm i\,W_\mu^2\right),\ W_\mu^3$

$U(1)_Y$: generator Y

gauge field B_μ

Free Lagrangian of (still massless) fermions:

$$\mathcal{L}_{0,\mathrm{ferm}} = \mathrm{i}\overline{\psi_f}\not{\partial}\psi_f = \mathrm{i}\overline{\Psi_L^{\mathrm{L}}}\not{\partial}\Psi_L^{\mathrm{L}} + \mathrm{i}\overline{\Psi_Q^{\mathrm{L}}}\not{\partial}\Psi_Q^{\mathrm{L}} + \mathrm{i}\overline{\psi_l^{\mathrm{R}}}\not{\partial}\psi_l^{\mathrm{R}} + \mathrm{i}\overline{\psi_u^{\mathrm{R}}}\not{\partial}\psi_u^{\mathrm{R}} + \mathrm{i}\overline{\psi_d^{\mathrm{R}}}\not{\partial}\psi_d^{\mathrm{R}}$$

Minimal substitution:

$$\partial_\mu \;\to\; D_\mu = \partial_\mu - \mathrm{i}g_2 T_{\mathrm{I}}^a W_\mu^a + \mathrm{i}g_1 \tfrac{1}{2} Y B_\mu \;=\; D_\mu^{\mathrm{L}}\omega_- + D_\mu^{\mathrm{R}}\omega_+,$$

$$D_\mu^{\mathrm{L}} = \partial_\mu - \frac{\mathrm{i}g_2}{\sqrt{2}}\begin{pmatrix} 0 & W_\mu^+ \\ W_\mu^- & 0 \end{pmatrix} - \frac{\mathrm{i}}{2}\begin{pmatrix} g_2 W_\mu^3 - g_1 Y^{\mathrm{L}} B_\mu & 0 \\ 0 & -g_2 W_\mu^3 - g_1 Y^{\mathrm{L}} B_\mu \end{pmatrix},$$

$$D_\mu^{\mathrm{R}} = \partial_\mu + \mathrm{i}g_1 \tfrac{1}{2} Y^{\mathrm{R}} B_\mu$$

Photon identification:

" rotation":

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} c_{\mathrm{W}} & s_{\mathrm{W}} \\ -s_{\mathrm{W}} & c_{\mathrm{W}} \end{pmatrix}\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix},$$

$c_{\mathrm{W}} = \cos\theta_{\mathrm{W}}$, $s_{\mathrm{W}} = \sin\theta_{\mathrm{W}}$,
θ_{W} = mixing angle

$$D_\mu^{\mathrm{L}}\Big|_{A_\mu} = -\frac{\mathrm{i}}{2}A_\mu \begin{pmatrix} -g_2 s_{\mathrm{W}} - g_1 c_{\mathrm{W}} Y^{\mathrm{L}} & 0 \\ 0 & g_2 s_{\mathrm{W}} - g_1 c_{\mathrm{W}} Y^{\mathrm{L}} \end{pmatrix} \overset{!}{=} \mathrm{i}eA_\mu \begin{pmatrix} Q_1 & 0 \\ 0 & Q_2 \end{pmatrix}$$

- charged difference in doublet $Q_1 - Q_2 = 1$ $\quad\to\quad$ $g_2 = \dfrac{e}{s_{\mathrm{W}}}$
- normalize $Y^{\mathrm{L/R}}$ such that $g_1 = \dfrac{e}{c_{\mathrm{W}}}$

$\hookrightarrow$ Y fixed by "Gell-Mann–Nishijima relation": $Q = T_{\mathrm{I}}^3 + \dfrac{Y}{2}$

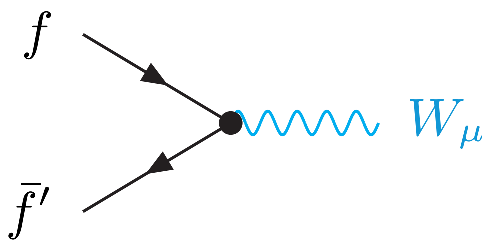
Fermion–gauge-boson interaction:

$$\mathcal{L}_{\mathrm{ferm,YM}} = \frac{e}{\sqrt{2}s_{\mathrm{W}}}\overline{\Psi^{\mathrm{L}}_F}\begin{pmatrix} 0 & \not{W}^+ \\ \not{W}^- & 0 \end{pmatrix}\Psi^{\mathrm{L}}_F + \frac{e}{2c_{\mathrm{W}}s_{\mathrm{W}}}\overline{\Psi^{\mathrm{L}}_F}\sigma^3\not{Z}\Psi^{\mathrm{L}}_F$$

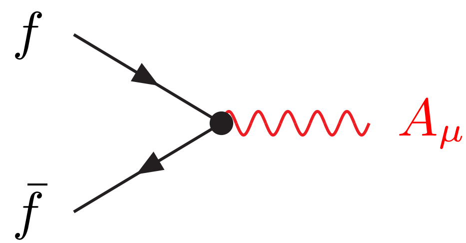
$$- e\frac{s_{\mathrm{W}}}{c_{\mathrm{W}}}Q_f\overline{\psi_f}\not{Z}\psi_f - eQ_f\overline{\psi_f}\not{A}\psi_f$$

(f=all fermions, F= all doublets)

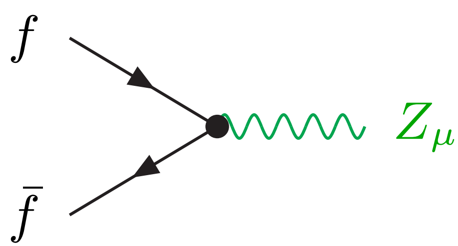
Feynman rules:



$$\frac{\mathrm{i}e}{\sqrt{2}s_{\mathrm{W}}}\gamma_\mu\omega_-$$



$$-\mathrm{i}Q_f e\gamma_\mu$$



$$\mathrm{i}e\gamma_\mu(g_f^+\omega_+ + g_f^-\omega_-) = \mathrm{i}e\gamma_\mu(v_f - a_f\gamma_5)$$

with $g_f^+ = -\frac{s_{\mathrm{W}}}{c_{\mathrm{W}}}Q_f, \quad g_f^- = -\frac{s_{\mathrm{W}}}{c_{\mathrm{W}}}Q_f + \frac{T^3_{\mathrm{I},f}}{c_{\mathrm{W}}s_{\mathrm{W}}},$

$$v_f = -\frac{s_{\mathrm{W}}}{c_{\mathrm{W}}}Q_f + \frac{T^3_{\mathrm{I},f}}{2c_{\mathrm{W}}s_{\mathrm{W}}}, \quad a_f = \frac{T^3_{\mathrm{I},f}}{2c_{\mathrm{W}}s_{\mathrm{W}}}$$

gauge field Lagrangian (Yang-Mills Lagrangian)

$$\mathcal{L}_{\rm YM} = -\frac{1}{4} W^a_{\mu\nu} W^{a,\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

Field-strength tensors:

$$W^a_{\mu\nu} \;=\; \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g_2 \epsilon^{abc} W^b_\mu W^c_\nu, \qquad B_{\mu\nu} \;=\; \partial_\mu B_\nu - \partial_\nu B_\mu$$

Lagrangian in terms of “physical” fields:

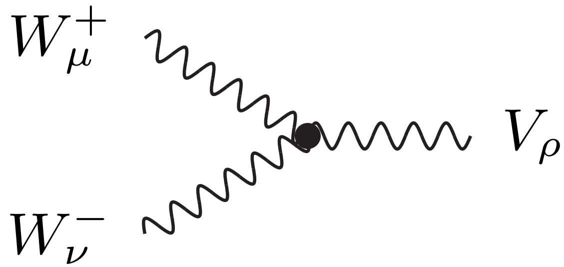
$$\begin{aligned}\mathcal{L}_{\rm YM} = &-\frac{1}{2}(\partial_\mu W^+_\nu - \partial_\nu W^+_\mu)(\partial^\mu W^{-,\nu} - \partial^\nu W^{-,\mu}) \\ &- \frac{1}{4}(\partial_\mu Z_\nu - \partial_\nu Z_\mu)(\partial^\mu Z^\nu - \partial^\nu Z^\mu) - \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu)\end{aligned}$$

$+$ (trilinear interaction terms involving AW^+W^-, ZW^+W^-)

$+$ (quadrilinear interaction terms involving AAW^+W^-, AZW^+W^-, ZZW^+W^-, $W^+W^-W^+W^-$)

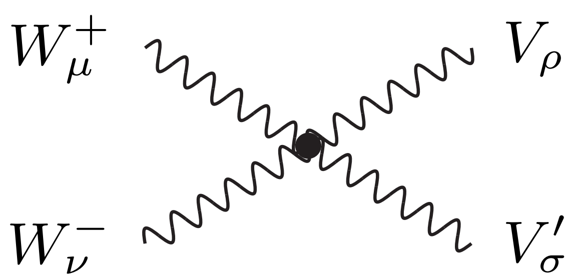
Feynman rules for gauge-boson self-interactions:

(fields and momenta incoming)



$$\mathrm{i}eC_{WWV}\Big[g_{\mu\nu}(k_+ - k_-)_\rho + g_{\nu\rho}(k_- - k_V)_\mu + g_{\rho\mu}(k_V - k_+)_\nu\Big]$$

with $C_{WW\gamma} = 1, \quad C_{WWZ} = -\frac{c_\mathrm{W}}{s_\mathrm{W}}$



$$\mathrm{i}e^2C_{WWVV'}\Big[2g_{\mu\nu}g_{\rho\sigma} - g_{\mu\rho}g_{\sigma\nu} - g_{\mu\sigma}g_{\nu\rho}\Big]$$

with $C_{WW\gamma\gamma} = -1, \qquad C_{WW\gamma Z} = \frac{c_\mathrm{W}}{s_\mathrm{W}},$

$$C_{WWZZ} = -\frac{c_\mathrm{W}^2}{s_\mathrm{W}^2}, \quad C_{WWWW} = \frac{1}{s_\mathrm{W}^2}$$

Higgs mechanism $\Rightarrow$ masses of W and Z bosons

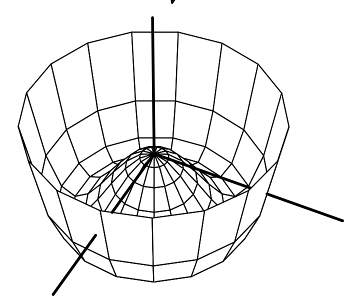
spontaneous breaking $SU(2)_I \times U(1)_Y \rightarrow U(1)_Q$
unbroken em. gauge symmetry, massless photon

Minimal scalar sector with complex scalar doublet $\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$, $Y_\Phi = 1$

Scalar self-interaction via Higgs potential:

$$V(\Phi) = -\mu^2 \Phi^\dagger \Phi + \frac{\lambda}{4}(\Phi^\dagger \Phi)^2, \quad \mu^2, \lambda > 0,$$

$= \mathrm{SU(2)_I \times U(1)_Y}$ symmetric



$V(\Phi) =$ minimal for $|\Phi| = \sqrt{\frac{2\mu^2}{\lambda}} \equiv \frac{v}{\sqrt{2}} > 0$

ground state Φ_0 (=vacuum expectation value of Φ) not unique

specific choice $\Phi_0 = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$ not gauge invariant $\Rightarrow$ spontaneous symmetry breaking

elmg. gauge invariance unbroken, since $Q\Phi_0 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \Phi_0 = 0$

Field excitations in Φ:

$$\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \frac{1}{\sqrt{2}}\Big(v + H(x) + \mathrm{i}\chi(x)\Big) \end{pmatrix}$$

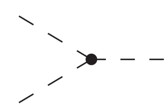
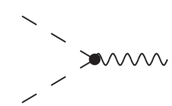
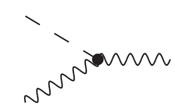
Gauge-invariant Lagrangian of Higgs sector: $\quad (\phi^- = (\phi^+)^\dagger)$

$$\mathcal{L}_\mathrm{H} = (D_\mu\Phi)^\dagger(D^\mu\Phi) - V(\Phi) \qquad \text{with } D_\mu = \partial_\mu - \mathrm{i}g_2\frac{\sigma^a}{2}W^a_\mu + \mathrm{i}\frac{g_1}{2}B_\mu$$

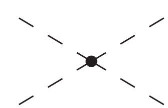
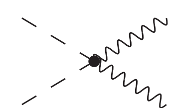
$$= (\partial_\mu\phi^+)(\partial^\mu\phi^-) - \frac{\mathrm{i}ev}{2s_\mathrm{W}}(W^+_\mu\partial^\mu\phi^- - W^-_\mu\partial^\mu\phi^+) + \frac{e^2v^2}{4s^2_\mathrm{W}}W^+_\mu W^{-,\mu}$$

$$+\frac{1}{2}(\partial\chi)^2 + \frac{ev}{2c_\mathrm{W}s_\mathrm{W}}Z_\mu\partial^\mu\chi + \frac{e^2v^2}{4c^2_\mathrm{W}s^2_\mathrm{W}}Z^2 + \frac{1}{2}(\partial H)^2 - \mu^2H^2$$

$+$ (trilinear SSS, SSV, SVV interactions)

$+$ (quadrilinear $SSSS$, $SSVV$ interactions)

Implications:

- gauge-boson masses: $\quad M_\mathrm{W} = \frac{ev}{2s_\mathrm{W}}, \quad M_\mathrm{Z} = \frac{ev}{2c_\mathrm{W}s_\mathrm{W}} = \frac{M_\mathrm{W}}{c_\mathrm{W}}, \quad M_\gamma = 0$
- physical Higgs boson H: $\quad M_\mathrm{H} = \sqrt{2\mu^2}$ = free parameter
- would-be Goldstone bosons $\phi^\pm$, χ: $\quad$ unphysical degrees of freedom

Fermion masses

fermions in chiral representations of gauge symmetry

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L , \quad e_R \qquad \Rightarrow \text{mass term } m_e(\overline{e}_L e_R + \overline{e}_R e_L) = m_e \, \overline{e} e$$

not gauge invariant

solution of the SM: introduce Yukawa interaction

= new interaction of fermions with the Higgs field

gauge invariant interaction, g = Yukawa coupling constant

$$\mathcal{L}_{\text{Yuk}} = g \, [\, \overline{\psi^L} \, \Phi \, e_R \, + \, \overline{e_R} \, \Phi^\dagger \psi^L \,]$$

most transparent in unitary gauge

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \to \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H \end{pmatrix}$$

apply to the first lepton generation $\psi^L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \quad e_R$:

$$\frac{g}{\sqrt{2}} \left[(\overline{\nu_L}, \overline{e_L}) \begin{pmatrix} 0 \\ v + H \end{pmatrix} e_R + \overline{e_R}\,(0, v + H) \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \right]$$

$$= \underbrace{\frac{g}{\sqrt{2}} v}_{m_e} \left[\overline{e_L} e_R + \overline{e_R} e_L\right] + \frac{g}{\sqrt{2}} H \left[\overline{e_L} e_R + \overline{e_R} e_L\right]$$

$$= m_e\, \overline{e} e + \frac{m_e}{v} H\, \overline{e} e$$

3 generations of leptons and quarks

Lagrangian for Yukawa couplings:

$$\mathcal{L}_{\rm Yuk} \;=\; -\overline{\Psi^{\rm L}_L} G_l \psi^{\rm R}_l \Phi - \overline{\Psi^{\rm L}_Q} G_u \psi^{\rm R}_u \tilde{\Phi} - \overline{\Psi^{\rm L}_Q} G_d \psi^{\rm R}_d \Phi + \text{h.c.}$$

- G_l, G_u, G_d = 3×3 matrices in 3-dim. space of generations (ν masses ignored)
- $\tilde{\Phi} = i\sigma^2 \Phi^* = \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix}$ = charge conjugate Higgs doublet, $Y_{\tilde{\Phi}} = -1$

Fermion mass terms:

mass terms = bilinear terms in $\mathcal{L}_{\rm Yuk}$, obtained by setting $\Phi \to \Phi_0$:

$$\mathcal{L}_{m_f} \;=\; -\frac{v}{\sqrt{2}} \overline{\psi^{\rm L}_l} G_l \psi^{\rm R}_l - \frac{v}{\sqrt{2}} \overline{\psi^{\rm L}_u} G_u \psi^{\rm R}_u - \frac{v}{\sqrt{2}} \overline{\psi^{\rm L}_d} G_d \psi^{\rm R}_d + \text{h.c.}$$

$\hookrightarrow$ diagonalization by unitary field transformations $\quad (f = l, u, d)$

$\hat{\psi}^{\rm L/R}_f \equiv U^{\rm L/R}_f \psi^{\rm L/R}_f \quad$ such that $\quad \frac{v}{\sqrt{2}} U^{\rm L}_f G_f (U^{\rm R}_f)^\dagger = {\rm diag}(m_f)$

$\Rightarrow$ standard form: $\quad \mathcal{L}_{m_f} \;=\; -m_f \overline{\hat{\psi}^{\rm L}_f} \hat{\psi}^{\rm R}_f + \text{h.c.} \;=\; -m_f \overline{\hat{\psi}_f} \hat{\psi}_f$

Quark mixing:

- ψ_f correspond to eigenstates of the gauge interaction
- $\hat{\psi}_f$ correspond to mass eigenstates,

  for massless neutrinos define $\quad \hat{\psi}^{\mathrm{L}}_{\nu} \equiv U^{\mathrm{L}}_{l} \psi^{\mathrm{L}}_{\nu} \quad \rightarrow$ no lepton-flavour changing

Yukawa and gauge interactions in terms of mass eigenstates:

$$\mathcal{L}_{\mathrm{Yuk}} = -\frac{\sqrt{2}m_l}{v}\left(\phi^+ \overline{\hat{\psi}^{\mathrm{L}}_{\nu_l}} \hat{\psi}^{\mathrm{R}}_l + \phi^- \overline{\hat{\psi}^{\mathrm{R}}_l} \hat{\psi}^{\mathrm{L}}_{\nu_l}\right) + \frac{\sqrt{2}m_u}{v}\left(\phi^+ \overline{\hat{\psi}^{\mathrm{R}}_u} V \hat{\psi}^{\mathrm{L}}_d + \phi^- \overline{\hat{\psi}^{\mathrm{L}}_d} V^\dagger \hat{\psi}^{\mathrm{R}}_u\right)$$
$$- \frac{\sqrt{2}m_d}{v}\left(\phi^+ \overline{\hat{\psi}^{\mathrm{L}}_u} V \hat{\psi}^{\mathrm{R}}_d + \phi^- \overline{\hat{\psi}^{\mathrm{R}}_d} V^\dagger \hat{\psi}^{\mathrm{L}}_u\right) - \frac{m_f}{v} \mathrm{i}\, \mathrm{sgn}(T^3_{\mathrm{I},f}) \chi \overline{\hat{\psi}_f} \gamma_5 \hat{\psi}_f$$
$$- \frac{m_f}{v}(v+H)\, \overline{\hat{\psi}_f} \hat{\psi}_f ,$$

$$\mathcal{L}_{\mathrm{ferm,YM}} = \frac{e}{\sqrt{2}s_{\mathrm{W}}} \overline{\hat{\Psi}^{\mathrm{L}}_L} \begin{pmatrix} 0 & \not{W}^+ \\ \not{W}^- & 0 \end{pmatrix} \hat{\psi}^{\mathrm{L}}_L + \frac{e}{\sqrt{2}s_{\mathrm{W}}} \overline{\hat{\Psi}^{\mathrm{L}}_Q} \begin{pmatrix} 0 & V\not{W}^+ \\ V^\dagger \not{W}^- & 0 \end{pmatrix} \hat{\psi}^{\mathrm{L}}_Q$$
$$+ \frac{e}{2c_{\mathrm{W}}s_{\mathrm{W}}} \overline{\hat{\Psi}^{\mathrm{L}}_F} \sigma^3 \not{Z} \hat{\Psi}^{\mathrm{L}}_F - e\frac{s_{\mathrm{W}}}{c_{\mathrm{W}}} Q_f \overline{\hat{\psi}_f} \not{Z} \hat{\psi}_f - eQ_f \overline{\hat{\psi}_f} \not{A} \hat{\psi}_f$$

- only charged-current coupling of quarks modified by $V = U^{\mathrm{L}}_u (U^{\mathrm{L}}_d)^\dagger$ = unitary

  (Cabibbo–Kobayashi–Maskawa (CKM) matrix)
- Higgs–fermion coupling strength = $\frac{m_f}{v}$

Features of the CKM mixing:

- V = 3-dim. generalization of Cabibbo matrix U_C
- V is parametrized by 4 free parameters: 3 real angles, 1 complex phase
  $\hookrightarrow$ complex phase is the only source of CP violation in SM

  counting:

  $$\begin{pmatrix}\text{\#real d.o.f.}\\ \text{in } V\end{pmatrix} - \begin{pmatrix}\text{\#unitarity}\\ \text{relations}\end{pmatrix} - \begin{pmatrix}\text{\#phase diffs. of}\\ u\text{-type quarks}\end{pmatrix} - \begin{pmatrix}\text{\#phase diffs. of}\\ d\text{-type quarks}\end{pmatrix} - \begin{pmatrix}\text{\#phase diff. between}\\ u\text{- and } d\text{-type quarks}\end{pmatrix}$$

  $$= 18 - 9 - 2 - 2 - 1 = 4$$
- no flavour-changing neutral currents in lowest order,
  flavour-changing suppressed by factors $G_\mu(m_{q_1}^2 - m_{q_2}^2)$ in higher orders
  ("Glashow–Iliopoulos–Maiani mechanism")