Field Theory and Standard Model. Part II

general gauge: Goldstone fields $\phi^{\pm}$, χ are present

required: gauge fixing term $\mathcal{L}_{fix}$

R_ξ gauge:

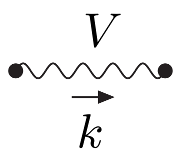
$$\mathcal{L}_{fix} = -\frac{1}{2\xi_\gamma}\left(F^\gamma\right)^2 - \frac{1}{2\xi_Z}\left(F^Z\right)^2 - \frac{1}{2\xi_W}\left(F^\pm\right)^2$$

with the gauge-fixing functionals F^a: (ξ_V = arbitrary gauge-fixing parameters)

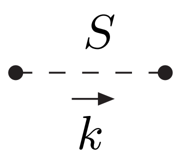
$$F^\pm = \partial W^\pm \mp \mathrm{i}\xi_\mathrm{W} M_\mathrm{W}\phi^\pm, \qquad F^\mathrm{Z} = \partial Z - \xi_\mathrm{Z} M_\mathrm{Z}\chi, \qquad F^\gamma = \partial A$$

- elimination of mixing terms $(W^{\pm}_{\mu}\partial^{\mu}\phi^{\mp})$, $(Z_{\mu}\partial^{\mu}\chi)$ in Lagrangian
  $\hookrightarrow$ decoupling of gauge and would-be Goldstone fields (no mix propagators)

- boson propagators:



$$D^{VV}_{\mu\nu}(k) = -\mathrm{i}\left[\frac{g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}}{k^2 - M_V^2} + \frac{k_\mu k_\nu}{k^2}\,\frac{\xi_V}{k^2 - \xi_V M_V^2}\right], \qquad V = W, Z, \gamma$$



$$D^{SS}(k) = \frac{\mathrm{i}}{k^2 - \xi_V M_V^2}, \qquad S = \phi, \chi$$

- important special cases:
  - $\xi_V = 1$: 't Hooft–Feynman gauge
    $\hookrightarrow$ convenient gauge-boson propagators $\frac{-\mathrm{i}g_{\mu\nu}}{k^2 - M_V^2}$
  - $\xi_{\mathrm{W}}, \xi_{\mathrm{Z}} \to \infty$: "unitary gauge"
    $\hookrightarrow$ elimination of would-be Goldstone bosons

Fermion masses

fermions in chiral representations of gauge symmetry

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \quad e_R \qquad \Rightarrow \text{ mass term } m_e(\overline{e}_L e_R + \overline{e}_R e_L) = m_e\,\overline{e}e$$

not gauge invariant

solution of the SM: introduce Yukawa interaction

= new interaction of fermions with the Higgs field

gauge invariant interaction, g = Yukawa coupling constant

$$\mathcal{L}_{\text{Yuk}} = g\,[\,\overline{\psi^L}\,\Phi\,e_R + \overline{e_R}\,\Phi^\dagger\psi^L\,]$$

most transparent in unitary gauge

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H \end{pmatrix}$$

apply to the first lepton generation $\psi^L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \quad e_R$:

$$\frac{g}{\sqrt{2}} \left[(\overline{\nu_L}, \overline{e_L}) \begin{pmatrix} 0 \\ v + H \end{pmatrix} e_R + \overline{e_R} \, (0, v + H) \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \right]$$

$$= \underbrace{\frac{g}{\sqrt{2}} v}_{m_e} \left[\overline{e_L} e_R + \overline{e_R} e_L \right] + \frac{g}{\sqrt{2}} H \left[\overline{e_L} e_R + \overline{e_R} e_L \right]$$

$$= m_e \, \overline{e} e + \frac{m_e}{v} H \, \overline{e} e$$

3 generations of leptons and quarks

Lagrangian for Yukawa couplings:

$$\mathcal{L}_{\rm Yuk} \;=\; -\overline{\Psi^{\rm L}_L} G_l \psi^{\rm R}_l \Phi - \overline{\Psi^{\rm L}_Q} G_u \psi^{\rm R}_u \tilde{\Phi} - \overline{\Psi^{\rm L}_Q} G_d \psi^{\rm R}_d \Phi + \text{h.c.}$$

- G_l, G_u, G_d = 3×3 matrices in 3-dim. space of generations (ν masses ignored)
- $\tilde{\Phi} = i\sigma^2 \Phi^* = \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix}$ = charge conjugate Higgs doublet, $Y_{\tilde{\Phi}} = -1$

Fermion mass terms:

mass terms = bilinear terms in $\mathcal{L}_{\rm Yuk}$, obtained by setting $\Phi \to \Phi_0$:

$$\mathcal{L}_{m_f} \;=\; -\frac{v}{\sqrt{2}} \overline{\psi^{\rm L}_l} G_l \psi^{\rm R}_l - \frac{v}{\sqrt{2}} \overline{\psi^{\rm L}_u} G_u \psi^{\rm R}_u - \frac{v}{\sqrt{2}} \overline{\psi^{\rm L}_d} G_d \psi^{\rm R}_d + \text{h.c.}$$

$\hookrightarrow$ diagonalization by unitary field transformations $\quad (f = l, u, d)$

$\hat{\psi}^{\rm L/R}_f \equiv U^{\rm L/R}_f \psi^{\rm L/R}_f$ $\quad$ such that $\quad \frac{v}{\sqrt{2}} U^{\rm L}_f G_f (U^{\rm R}_f)^\dagger = {\rm diag}(m_f)$

$\Rightarrow$ standard form: $\quad \mathcal{L}_{m_f} \;=\; -m_f \overline{\hat{\psi}^{\rm L}_f} \hat{\psi}^{\rm R}_f + \text{h.c.} \;=\; -m_f \overline{\hat{\psi}_f} \hat{\psi}_f$

Quark mixing:

- ψ_f correspond to eigenstates of the gauge interaction
- $\hat{\psi}_f$ correspond to mass eigenstates,

  for massless neutrinos define $\quad \hat{\psi}^{\rm L}_{\nu} \equiv U^{\rm L}_{l} \psi^{\rm L}_{\nu} \quad \rightarrow$ no lepton-flavour changing

Yukawa and gauge interactions in terms of mass eigenstates:

$$\mathcal{L}_{\rm Yuk} = -\frac{\sqrt{2} m_l}{v} \left(\phi^{+} \overline{\hat{\psi}^{\rm L}_{\nu_l}} \hat{\psi}^{\rm R}_{l} + \phi^{-} \overline{\hat{\psi}^{\rm R}_{l}} \hat{\psi}^{\rm L}_{\nu_l} \right) + \frac{\sqrt{2} m_u}{v} \left(\phi^{+} \overline{\hat{\psi}^{\rm R}_{u}} V \hat{\psi}^{\rm L}_{d} + \phi^{-} \overline{\hat{\psi}^{\rm L}_{d}} V^{\dagger} \hat{\psi}^{\rm R}_{u} \right)$$
$$- \frac{\sqrt{2} m_d}{v} \left(\phi^{+} \overline{\hat{\psi}^{\rm L}_{u}} V \hat{\psi}^{\rm R}_{d} + \phi^{-} \overline{\hat{\psi}^{\rm R}_{d}} V^{\dagger} \hat{\psi}^{\rm L}_{u} \right) - \frac{m_f}{v} {\rm i}\, {\rm sgn}(T^3_{{\rm I},f}) \chi \overline{\hat{\psi}_f} \gamma_5 \hat{\psi}_f$$
$$- \frac{m_f}{v} (v + H) \overline{\hat{\psi}_f} \hat{\psi}_f ,$$

$$\mathcal{L}_{\rm ferm,YM} = \frac{e}{\sqrt{2} s_{\rm W}} \overline{\hat{\Psi}^{\rm L}_{L}} \begin{pmatrix} 0 & \not{W}^{+} \\ \not{W}^{-} & 0 \end{pmatrix} \hat{\psi}^{\rm L}_{L} + \frac{e}{\sqrt{2} s_{\rm W}} \overline{\hat{\Psi}^{\rm L}_{Q}} \begin{pmatrix} 0 & V \not{W}^{+} \\ V^{\dagger} \not{W}^{-} & 0 \end{pmatrix} \hat{\psi}^{\rm L}_{Q}$$
$$+ \frac{e}{2 c_{\rm W} s_{\rm W}} \overline{\hat{\Psi}^{\rm L}_{F}} \sigma^3 \not{Z} \hat{\Psi}^{\rm L}_{F} - e \frac{s_{\rm W}}{c_{\rm W}} Q_f \overline{\hat{\psi}_f} \not{Z} \hat{\psi}_f - e Q_f \overline{\hat{\psi}_f} \not{A} \hat{\psi}_f$$

- only charged-current coupling of quarks modified by $V = U^{\rm L}_{u} (U^{\rm L}_{d})^{\dagger}$ = unitary

  (Cabibbo–Kobayashi–Maskawa (CKM) matrix)
- Higgs–fermion coupling strength = $\dfrac{m_f}{v}$

Features of the CKM mixing:

- V = 3-dim. generalization of Cabibbo matrix U_{C}
- V is parametrized by 4 free parameters: 3 real angles, 1 complex phase
  $\hookrightarrow$ complex phase is the only source of CP violation in SM

  counting:

  $$\begin{pmatrix}\text{\#real d.o.f.}\\ \text{in } V\end{pmatrix} - \begin{pmatrix}\text{\#unitarity}\\ \text{relations}\end{pmatrix} - \begin{pmatrix}\text{\#phase diffs. of}\\ u\text{-type quarks}\end{pmatrix} - \begin{pmatrix}\text{\#phase diffs. of}\\ d\text{-type quarks}\end{pmatrix} - \begin{pmatrix}\text{\#phase diff. between}\\ u\text{- and } d\text{-type quarks}\end{pmatrix}$$

  $$= 18 - 9 - 2 - 2 - 1 = 4$$
- no flavour-changing neutral currents in lowest order,
  flavour-changing suppressed by factors $G_\mu(m_{q_1}^2 - m_{q_2}^2)$ in higher orders
  (“Glashow–Iliopoulos–Maiani mechanism”)

6. Phenomenology of W and Z bosons and precision tests

Basic parameters and relations

ew mixing angle: $s_{\mathrm{W}} \equiv \sin\theta_{\mathrm{W}}, \quad c_{\mathrm{W}} \equiv \cos\theta_{\mathrm{W}}$

gauge coupling constants: $g_2 = \frac{e}{s_{\mathrm{W}}}, \quad g_1 = \frac{e}{c_{\mathrm{W}}}$

vector boson masses:

$$M_W = \tfrac{1}{2} g_2 v = \frac{ev}{2s_{\mathrm{W}}}$$

$$M_Z = \frac{ev}{2s_{\mathrm{W}}c_{\mathrm{W}}} = \frac{M_W}{c_{\mathrm{W}}}$$

$$s_{\mathrm{W}}^2 = 1 - \frac{M_W^2}{M_Z^2}$$

neutral current (NC) couplings:

$$a_f = \frac{g_2}{2c_{\mathrm{W}}}\, T_3^f$$

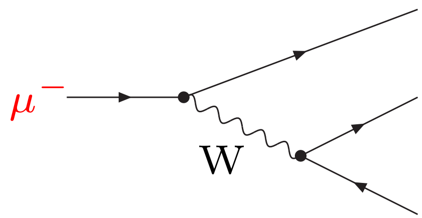
$$v_f = \frac{g_2}{2c_{\mathrm{W}}}\left(T_3^f - 2Q_f s_{\mathrm{W}}\right)$$

features of the ew Standard Model

- Higgs boson not yet found, all other particles confirmed
- good description of data
- consistent quantum field theory
  - in accordance with unitarity
  - renormalizable $\Rightarrow$ predictions at higher orders
- formal parameters: $g_2,\ g_1,\ v,\ \lambda,\ g_f,\ V_{\mathrm{CKM}}$
  physical parameters: $\alpha,\ M_W,\ M_Z,\ M_H,\ m_f,\ V_{\mathrm{CKM}}$

observables and experiments

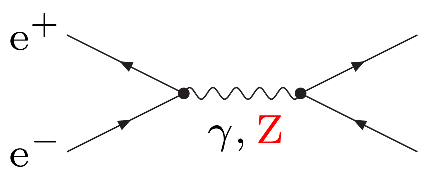
- Muon decay: $\mu^- \to \nu_\mu e^- \bar{\nu}_e$



determination of the Fermi constant

$$G_\mu = \frac{\pi\alpha M_Z^2}{\sqrt{2} M_W^2 (M_Z^2 - M_W^2)} + \dots$$

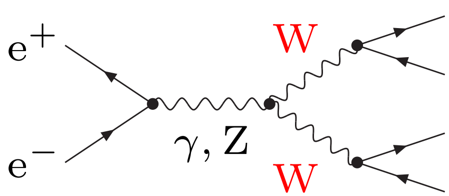
- Z production (LEP1/SLC): $e^+e^- \to Z \to f\bar{f}$

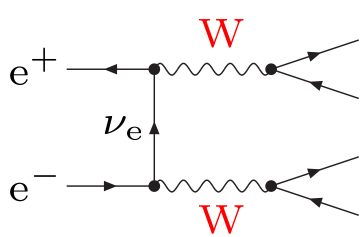


various precision measurements at the Z resonance: $M_Z, \Gamma_Z, \sigma_{had}, A_{FB}, A_{LR}$, etc.

$\Rightarrow$ good knowledge of the $Zf\bar{f}$ sector

- W-pair production (LEP2/ILC): $e^+e^- \to WW \to 4f(+\gamma)$





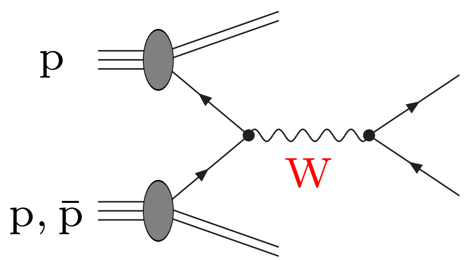
– measurement of M_W

– γWW/ZWW couplings

– quartic couplings: $\gamma\gamma WW$, γZWW

experiments at hadron colliders

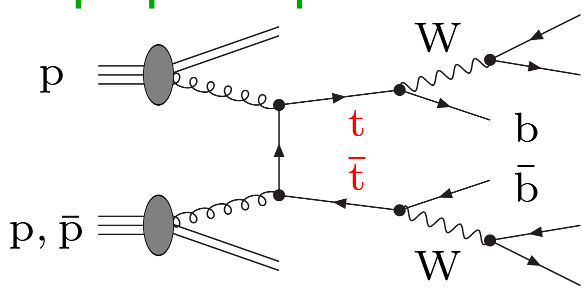
- W production (Tevatron/LHC): $\mathrm{pp}, \mathrm{p\bar{p}} \to \mathrm{W} \to l\nu_l(+\gamma)$



  – measurement of M_W

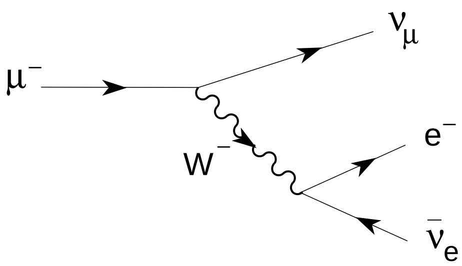
  – bounds on $\gamma\mathrm{WW}$ coupling

- top-quark production (Tevatron/LHC): $\mathrm{pp}, \mathrm{p\bar{p}} \to \mathrm{t\bar{t}} \to 6f$



  – measurement of m_t

μ decay



$$\mathcal{M} = \left(\frac{ig_2}{2\sqrt{2}}\right)^2 J_\rho^{(\mu)} \frac{-ig^{\rho\sigma}}{q^2 - M_W^2} J_\sigma^{(e)}$$

$$|q|^2 \simeq m_\mu^2 \ll M_W^2 : \qquad \mathcal{M} = -\frac{g_2^2}{8M_W^2} J_\rho^{(\mu)} J^{\rho\,(e)}$$

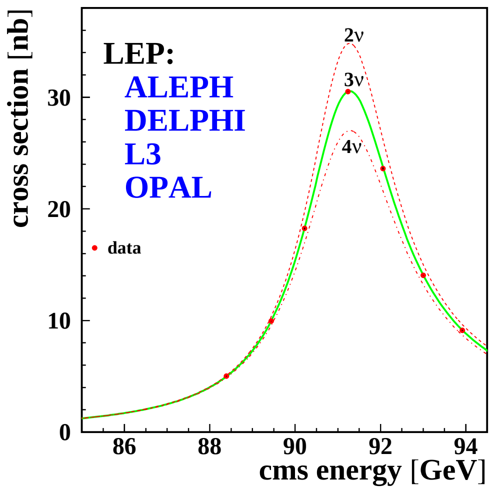
Fermi model *with point-like 4-fermion interaction:*

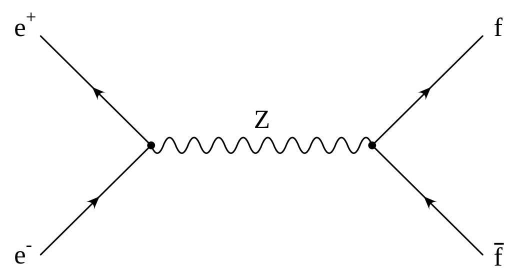
$$\mathcal{M} = \frac{G_F}{\sqrt{2}} J_\rho^{(\mu)} J^{\rho\,(e)}$$ *low-energy limit of SM*

$$\Rightarrow \quad \boxed{\frac{G_F}{\sqrt{2}} = \frac{g_2^2}{8M_W^2} = \frac{e^2}{8s_\mathrm{W}^2 c_\mathrm{W}^2 M_Z^2} = \frac{\pi\alpha}{2s_\mathrm{W}^2 c_\mathrm{W}^2 M_Z^2} = \frac{\pi\alpha}{2(1-M_W^2/M_Z^2)M_W^2}}$$

$$G_F = 1.16637(1)\cdot 10^{-5}\,\mathrm{GeV}^{-2}$$

Z resonance





$$\mathcal{M} = J_\mu^{(e)} \, \frac{-ig^{\mu\nu}}{s - M_Z^2 + iM_Z\Gamma_Z} \, J_\nu^{(f)}$$

propagator with finite width Γ_Z *(unstable particle)*

$$\Gamma_Z = \sum_f \Gamma(Z \to f\bar{f}), \quad \Gamma(Z \to f\bar{f}) = \frac{M_Z}{12\pi}\,(v_f^2 + a_f^2)$$

differential cross section at $s = M_Z^2$*:*

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} \sim (v_e^2 + a_e^2)(v_f^2 + a_f^2)\,(1 + \cos^2\theta) \;+\; (2v_e a_e)(2v_f a_f)\cdot 2\cos\theta$$

$\Rightarrow$ *forward-backward asymmetry* $A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = \frac{3}{4} A_e A_f$

$$A_f = \frac{2v_f a_f}{v_f^2 + a_f^2}$$

polarized cross section for $e^-_{L,R}$*:*

$\Rightarrow$ *left-right asymmetry* $A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = A_e$

asymmetries determine $\sin^2\theta_{\mathrm{W}}$

input from experiments

- LEP1/SLC: $e^+e^- \to Z \to f\bar{f}$
  LEP1: $\sim 4 \times 10^6$ events/experiment
  4 experiments (1989 – 1995)

- LEP2: $e^+e^- \to W^+W^-$
  $\mathcal{O}(10^4)$ W pairs (1996 – 2000)

- Tevatron: $q\bar{q}' \to W \to l\nu, q\bar{q}'$
  ($p\bar{p}$) $q\bar{q}' \to t\bar{t}$, $t \to W^+b \to \ldots$

- low-energy experiments (μ decay, νN scattering, νe scattering, atomic parity violation, ...)

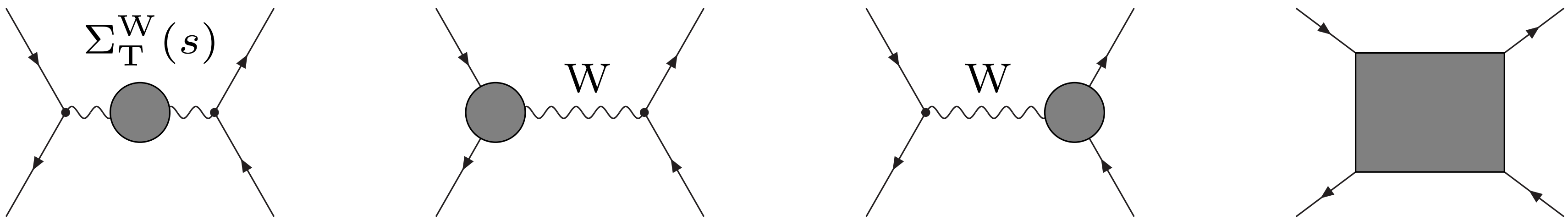
experimental results (selection)

M_Z [GeV]	$= 91.1875 \pm 0.0021$	0.002%
Γ_Z[GeV]	$= 2.4952 \pm 0.0023$	0.09%
$\sin^2\theta_\mathrm{eff}^\mathrm{lept}$	$= 0.23148 \pm 0.00017$	0.07%
M_W [GeV]	$= 80.398 \pm 0.025$	0.04%
m_t [GeV]	$= 173.1 \pm 1.3$	0.75%
G_F [GeV^{-2}]	$= 1.16637(1)10^{-5}$	0.001%

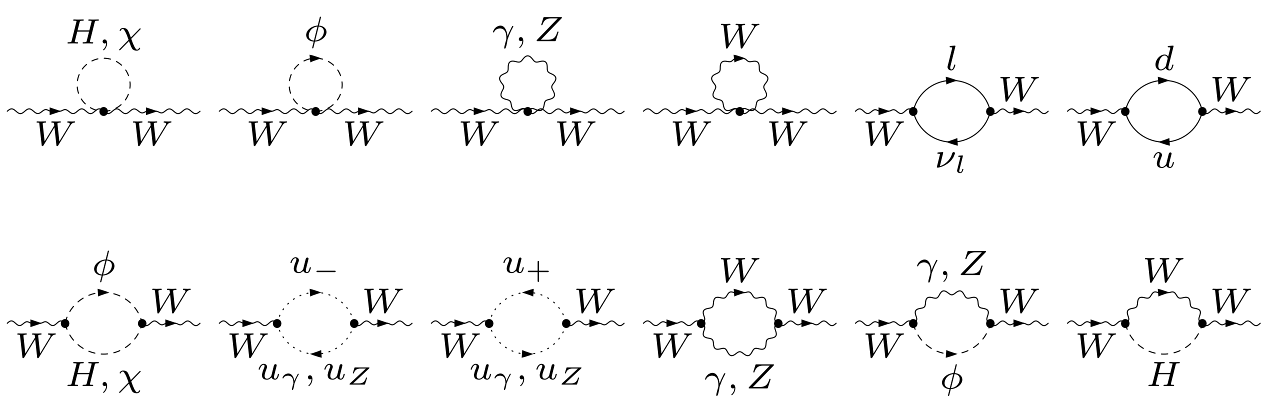
loop effects are at least one order of magnitude larger than experimental uncertainties

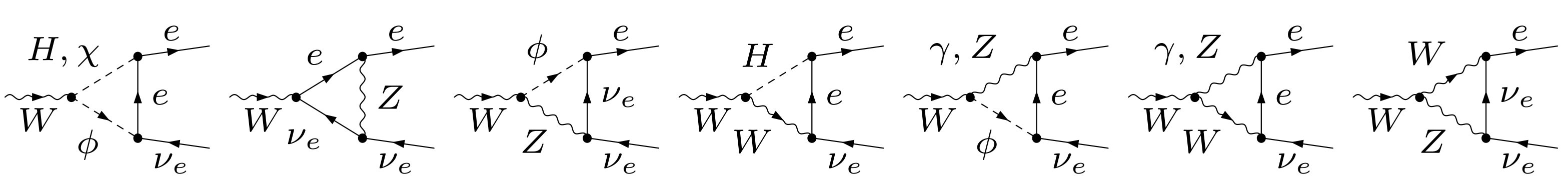
higher-order calculations are a necessity

example: 1-loop diagrams for μ decay amplitude

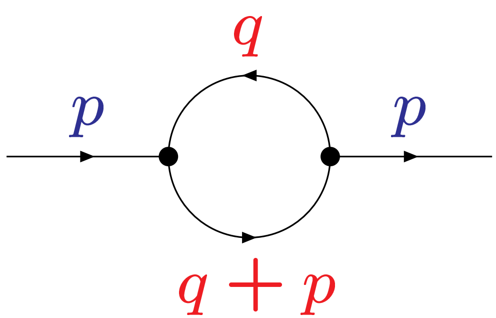


W self-energy $\quad$ $Wl\nu_l$ vertex correction $\quad$ box diagrams





Example of loop integral:



$$\sim \int d^4q \frac{1}{(q^2 - m_1^2)\left[(q+p)^2 - m_2^2\right]}$$

$$q \to \infty : \quad \sim \int^{\infty} \frac{q^3 dq}{q^4} = \int^{\infty} \frac{dq}{q} \quad \to \infty$$

$\Rightarrow$ integral diverges for large q

$\Rightarrow$ theory in this form not physically meaningful

needs (i) regularization

(ii) renormalization

Regularization:

theory modified such that expressions become mathematically meaningful

$\Rightarrow$ "regulator" introduced, removed at the end

e.g. cut-off in loop integral

$$\int_0^\infty d^4q \;\to \int_0^\Lambda d^4q; \quad \Lambda \to \infty \text{ at the end}$$

technically more convenient: dimensional regularization

$$\int d^4q \;\to \int d^Dq, \quad D = 4 - \varepsilon; \quad D \to 4 \text{ at the end}$$

Renormalization:

- absorption of divergencies
- determination of physical meaning of parameters order by order in perturbation theory

add <u>counterterms</u> that absorb divergent parts

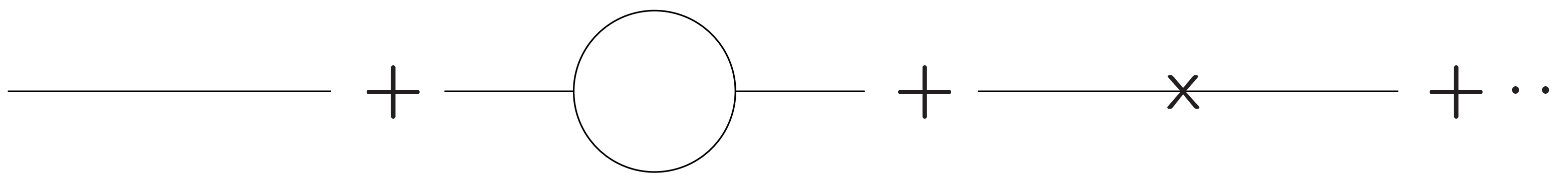
- parameters in $\mathcal{L}$ are formal, “bare parameters”

  $g_0 = g + \delta g$ *for a coupling,* $m_0 = m + \delta m$ *for a mass*
- $g,\, m$ are “physical”, *i.e.* measurable

mass renormalization, $m_0^2 = m^2 + \delta m^2$

Physical mass: pole of propagator

inverse propagator up to 1-loop order:

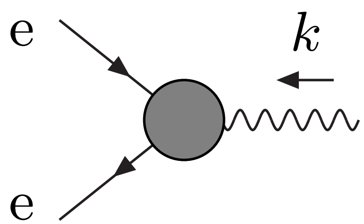


$p^2 - m^2$ $\qquad \Sigma(p^2) \qquad$ $-\,\delta m^2$

on-shell renormalization: $\delta m^2 = \mathrm{Re}\,\Sigma(m^2)$

charge renormalization: $e_0 = e + \delta e$

δe *cancels loop contributions to* $ee\gamma$ *vertex in the Thomson limit*



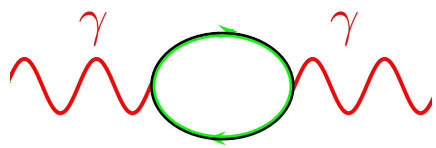
$A_\mu \xrightarrow{k \to 0} \mathrm{i}e\gamma_\mu$ for on-shell electrons

$\Rightarrow$ $e =$ elementary charge of classical electrodynamics

fine-structure constant $\alpha(0) = \dfrac{e^2}{4\pi} = 1/137.03599976$

δe *contains photon vacuum polarization* $\Pi^\gamma(k^2 = 0)$

photon vacuum polarization

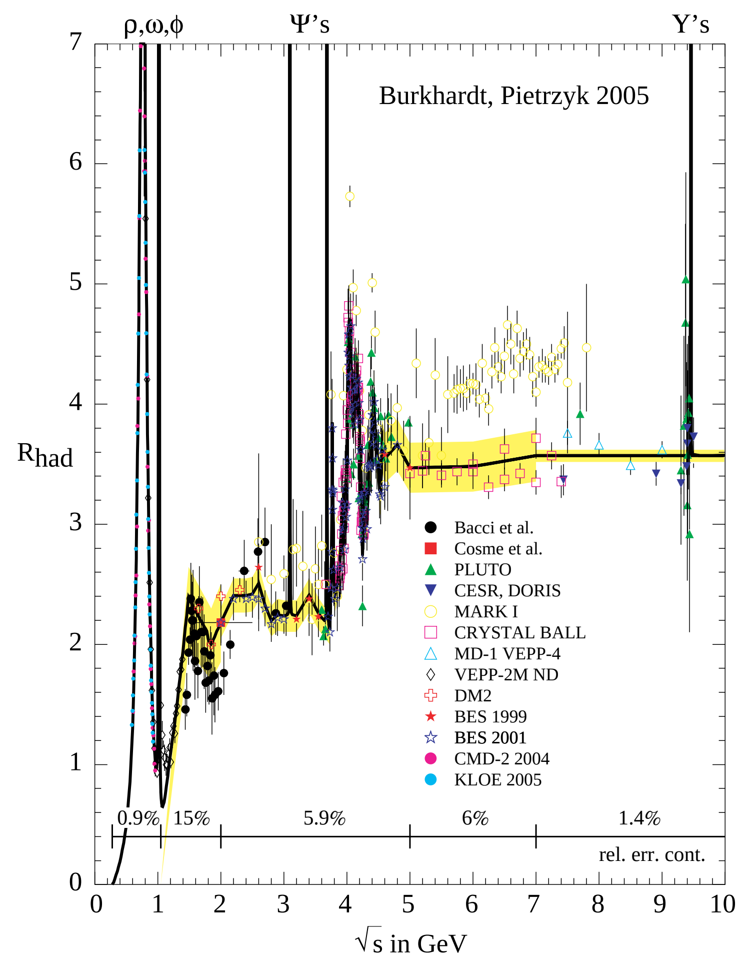


$$\Pi^\gamma(M_Z^2) - \Pi^\gamma(0) \equiv \Delta\alpha \quad \to \quad \alpha(M_Z) = \frac{\alpha}{1-\Delta\alpha}$$

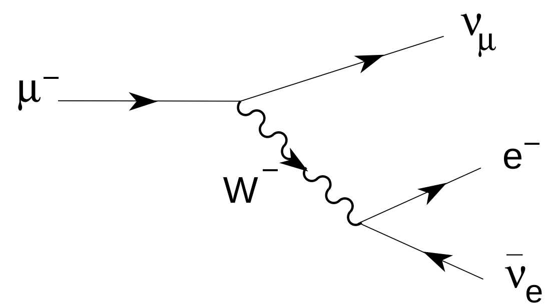
$$\begin{aligned}\Delta\alpha &= \Delta\alpha_{\rm lept} + \Delta\alpha_{\rm had}, \\ \Delta\alpha_{\rm lept} &= 0.031498 \quad (3-\text{loop}) \\ \Delta\alpha_{\rm had} &= 0.02758 \pm 0.00035\end{aligned}$$

$$\Delta\alpha_{\rm had} = -\frac{\alpha}{3\pi} M_Z^2 \,{\rm Re} \int_{4m_\pi^2}^{\infty} ds' \,\frac{R_{\rm had}(s')}{s'(s'-M_Z^2-i\epsilon)}$$

$$R_{\rm had} = \frac{\sigma(e^+e^- \to \gamma^* \to {\rm hadrons})}{\sigma(e^+e^- \to \gamma^* \to \mu^+\mu^-)}$$



$M_W - M_Z$ correlation



$$\frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{2M_W^2\left(1 - M_W^2/M_Z^2\right)}$$

with loop contributions

$$\frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{2M_W^2\left(1 - M_W^2/M_Z^2\right)} \cdot (1 + \Delta r)$$

Δr : quantum correction

$\Delta r = \Delta r(m_t, M_H)$

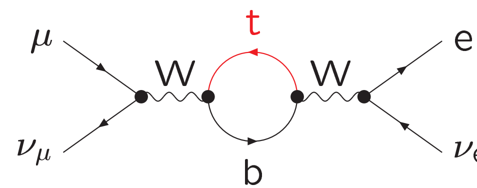
determines W mass

$$M_W = M_W(\alpha, G_F, M_Z, m_t, M_H)$$

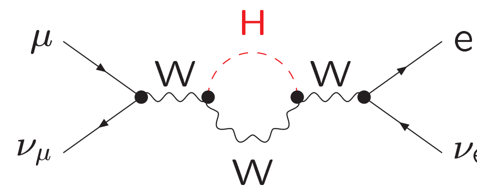
complete at 2-loop order

1-loop examples

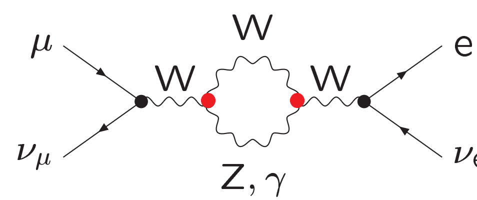
- top quark



- Higgs boson

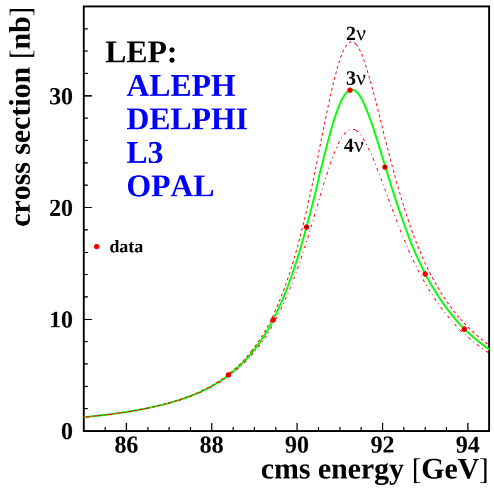


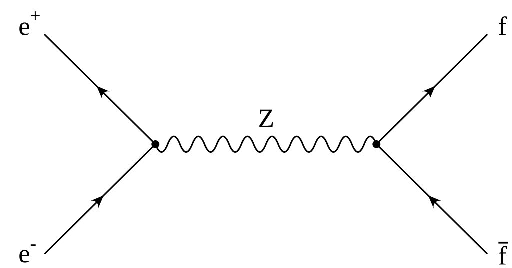
- gauge-boson self-couplings



full structure of SM

Z resonance





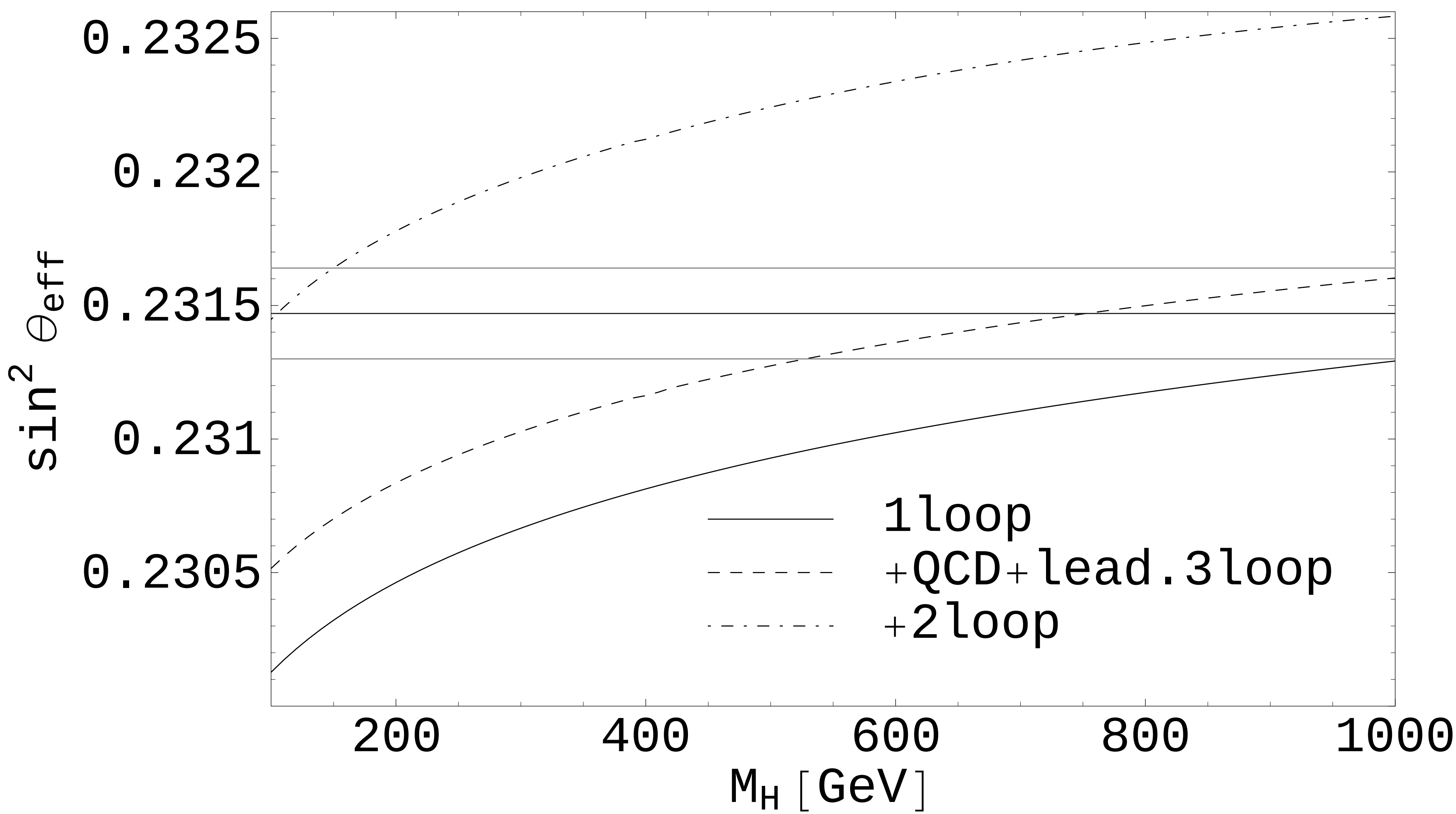
- effective Z boson couplings with higher-order $\Delta g_{V,A}$

$$v_f \to g_V^f = v_f + \Delta g_V^f, \qquad a_f \to g_A^f = a_f + \Delta g_A^f$$

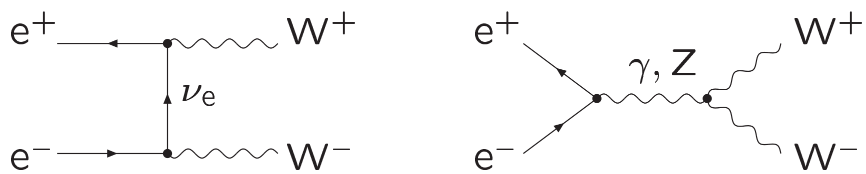
- effective ew mixing angle (for $f = e$):

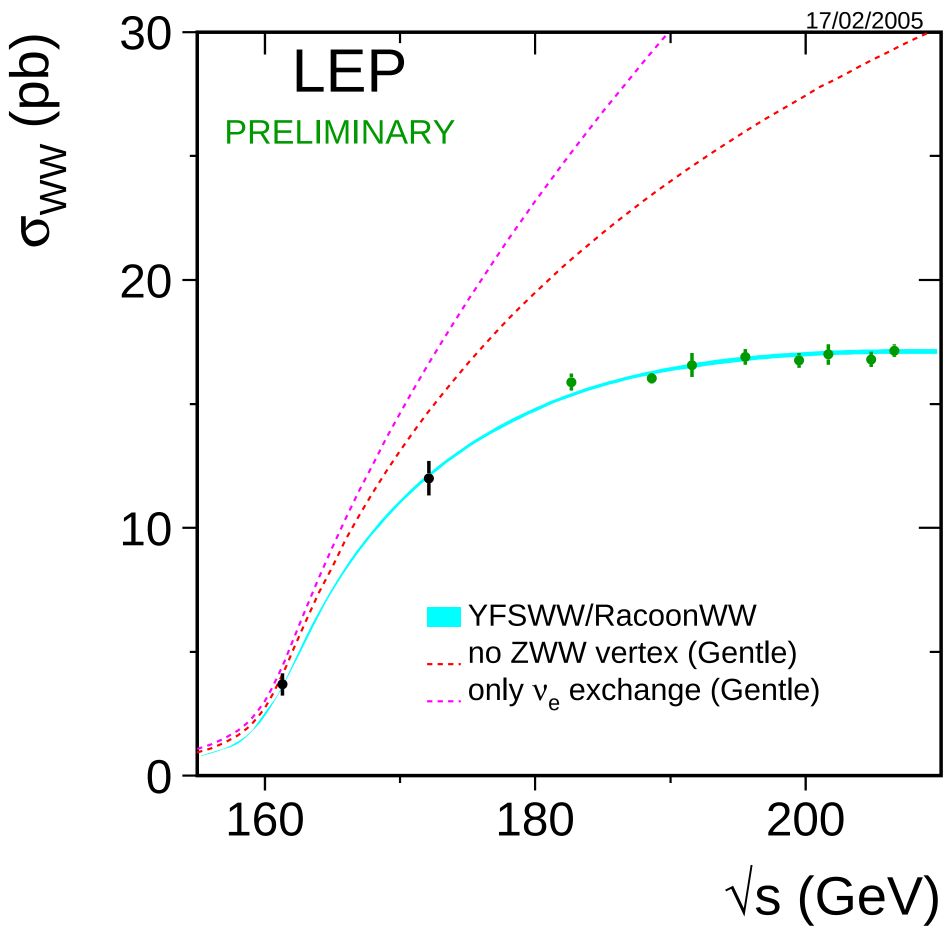
$$\sin^2\theta_{\text{eff}} = \frac{1}{4}\left(1 - \text{Re}\,\frac{g_V^e}{g_A^e}\right) = \kappa \cdot \left(1 - \frac{M_W^2}{M_Z^2}\right)$$

importance of two-loop calculations

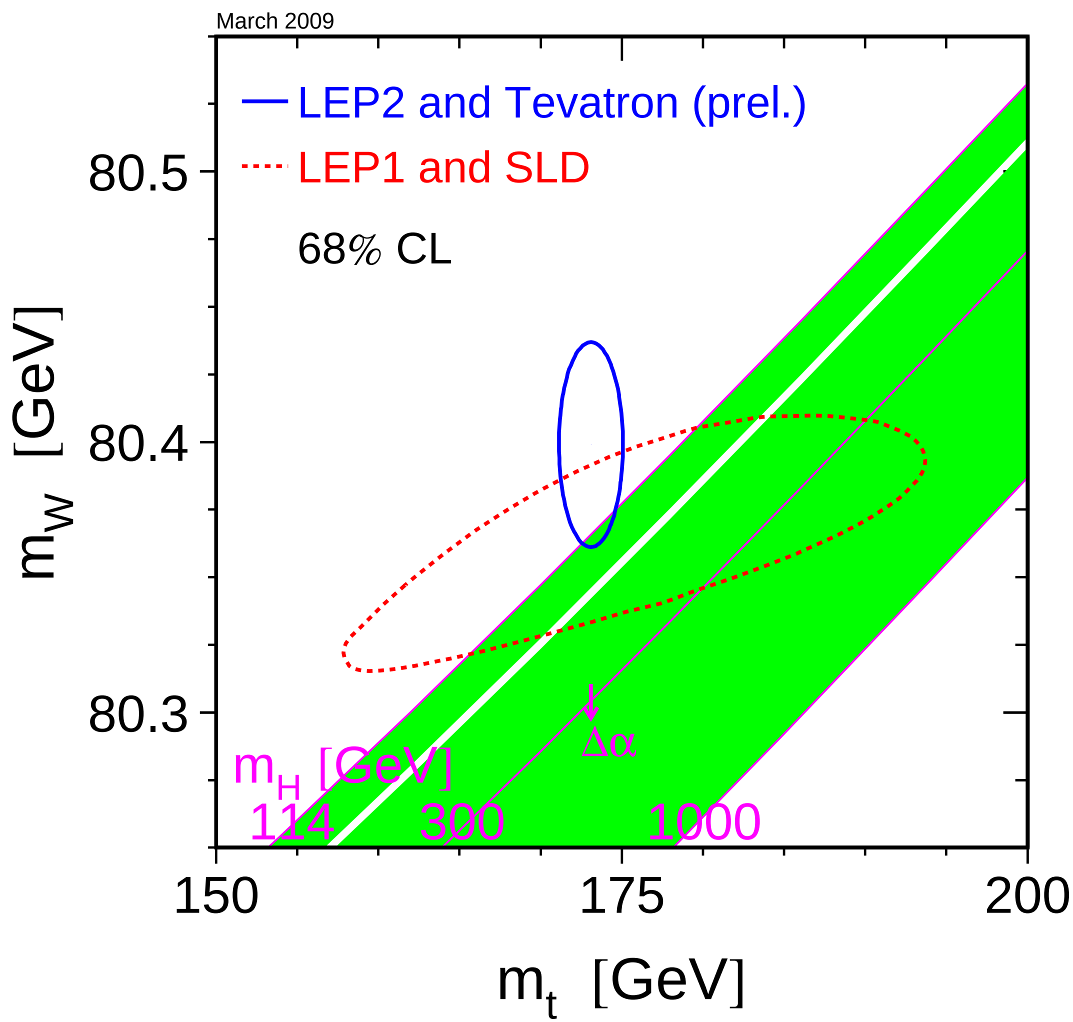


W-pair production

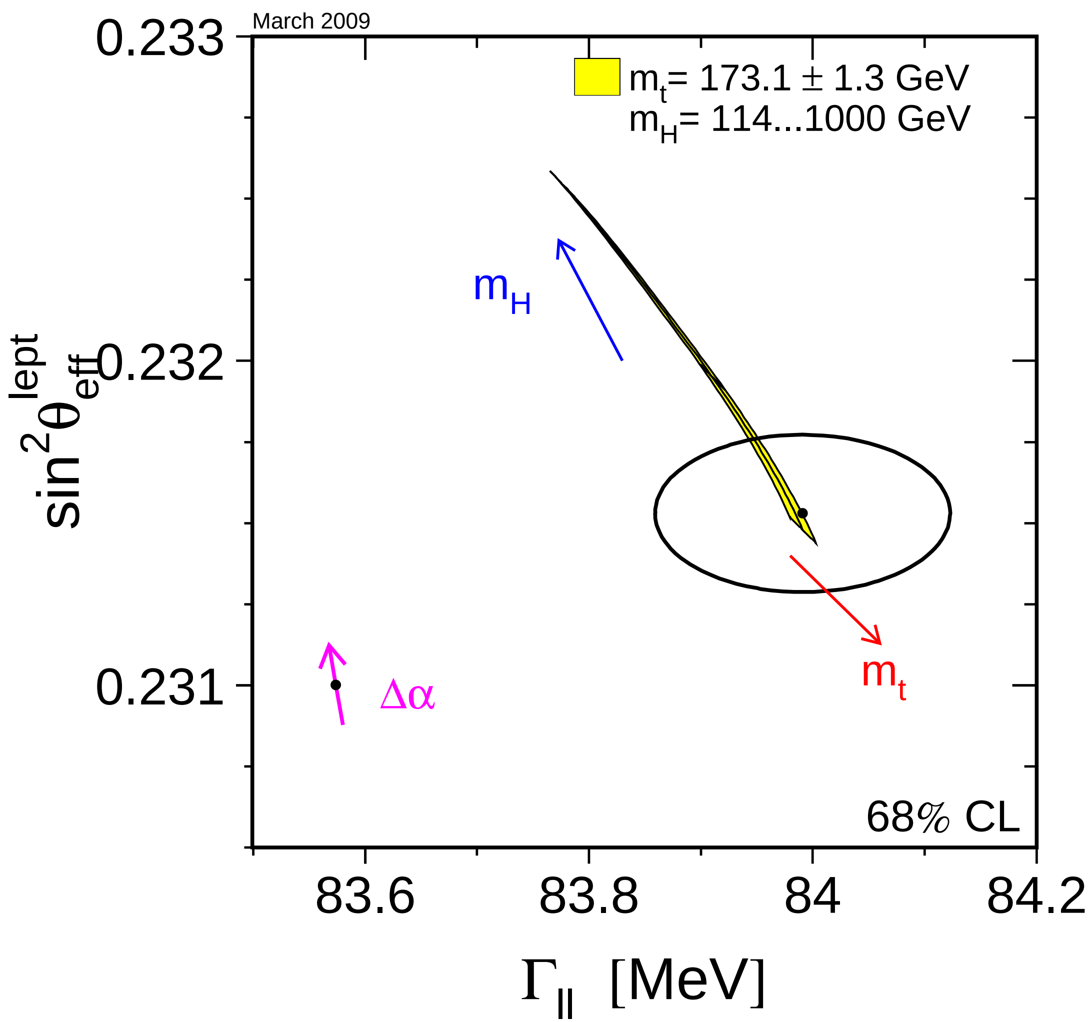


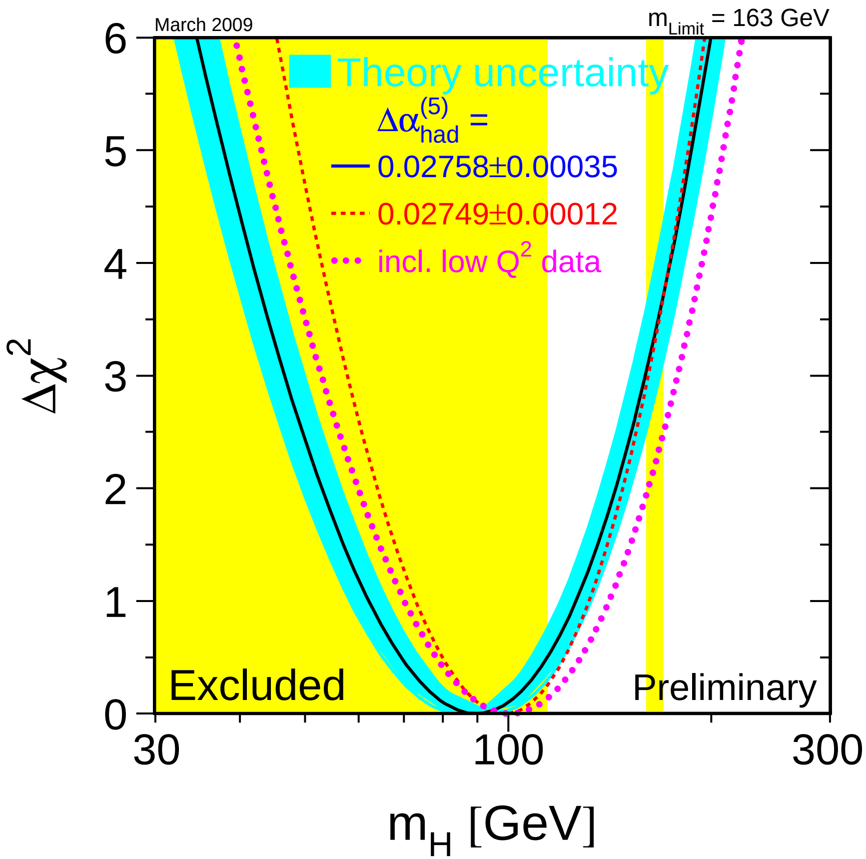


LEP Electroweak Working Group



LEP Electroweak Working Group





blueband: theory uncertainty

"Precision Calculations at the Z Resonance" CERN 95-03

[Bardin, Hollik, Passarino (eds.)]

$$M_{\rm H} < 163\ {\rm GeV} \quad (95\%{\rm C.L.})$$

with direct search $M_{\rm H} > 114$ GeV:

$$M_{\rm H} < 191\ {\rm GeV} \quad (95\%{\rm C.L.})$$