Field Theory and Standard Model. Part III

7. Higgs bosons

Higgs potential: $V = -\mu^2\ \left(\Phi^\dagger\Phi\right)^2 + \ \frac{\lambda}{4}\left(\Phi^\dagger\Phi\right)^4$

Higgs field in unitary gauge: $\Phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$

$H(x)$: *real scalar field, describes neutral spin-0 bosons*

minimum of V: $v = \frac{2\mu}{\sqrt{\lambda}}, \quad M_H = \mu\sqrt{2}$

$$\Rightarrow \quad \lambda = \frac{4\mu^2}{v^2} = \frac{2M_H^2}{v^2}$$

$$\boxed{V = \frac{M_H^2}{2}H^2 + \frac{M_H^2}{2v}H^3 + \frac{M_H^4}{8v^2}H^4}$$

M_H *is the only free parameter*

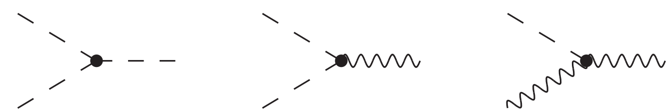
gauge invariant Lagrangian of the Higgs sector

$$\mathcal{L}_{\mathrm{H}} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi) \qquad \text{with } D_\mu = \partial_\mu - \mathrm{i} g_2 \frac{\sigma^a}{2} W_\mu^a + \mathrm{i} \frac{g_1}{2} B_\mu$$

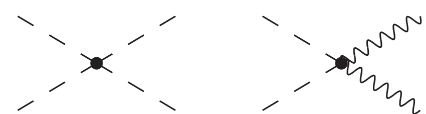
$$= (\partial_\mu \phi^+)(\partial^\mu \phi^-) - \frac{\mathrm{i} e v}{2 s_{\mathrm{W}}} (W_\mu^+ \partial^\mu \phi^- - W_\mu^- \partial^\mu \phi^+) + \frac{e^2 v^2}{4 s_{\mathrm{W}}^2} W_\mu^+ W^{-,\mu}$$

$$+ \frac{1}{2} (\partial \chi)^2 + \frac{e v}{2 c_{\mathrm{W}} s_{\mathrm{W}}} Z_\mu \partial^\mu \chi + \frac{e^2 v^2}{4 c_{\mathrm{W}}^2 s_{\mathrm{W}}^2} Z^2 + \frac{1}{2} (\partial H)^2 - \mu^2 H^2$$

$+$ (trilinear SSS, SSV, SVV interactions)



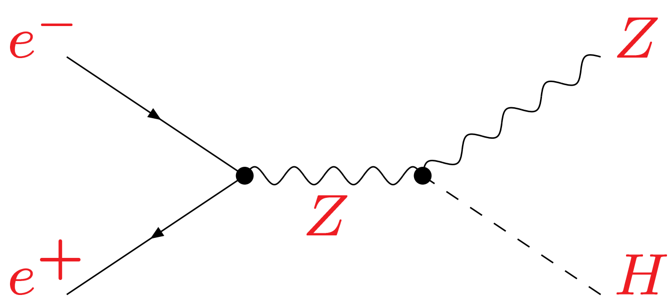
$+$ (quadrilinear $SSSS$, $SSVV$ interactions)



$\Rightarrow$ trilinear H-V-V interactions, V=W and Z

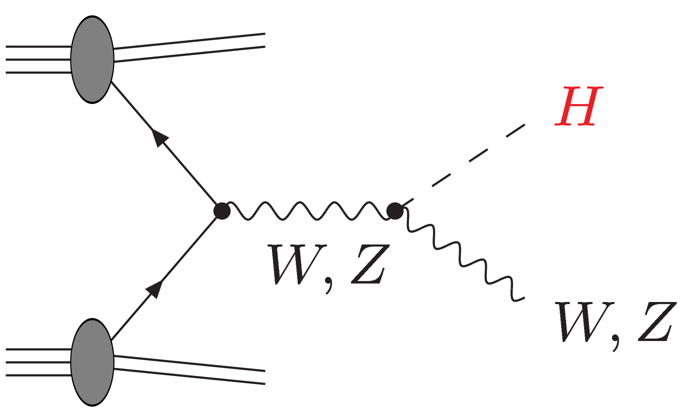
experimental search

LEP:



excluded $M_H < 114\,\text{GeV}$

Tevatron:



excluded $160 < M_H < 170\,\text{GeV}$

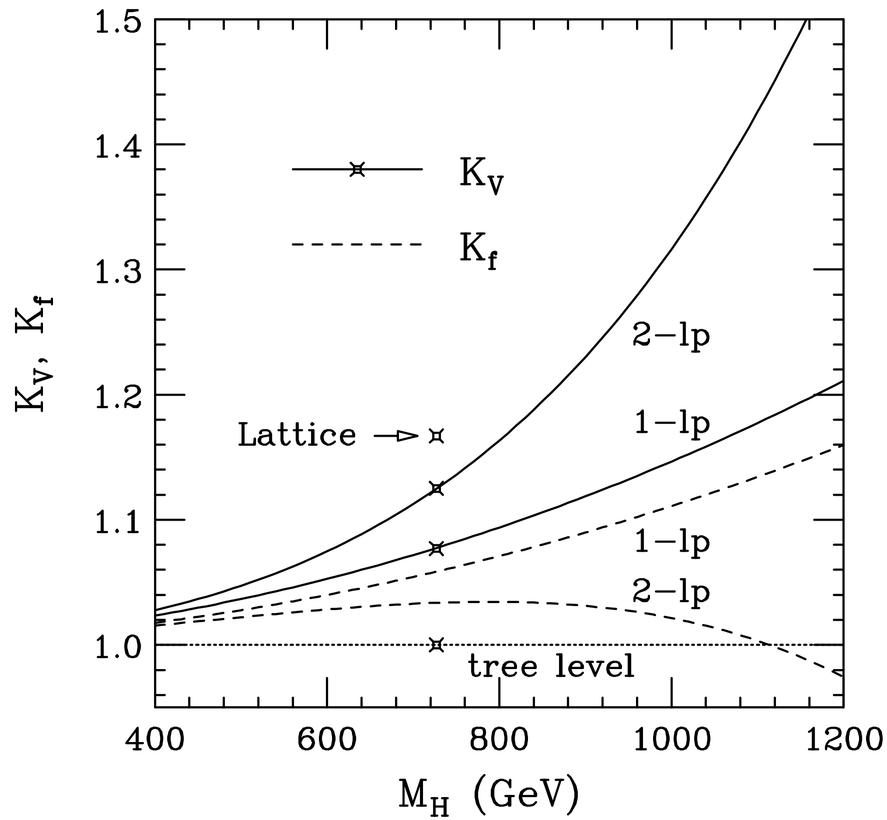
Theoretical bounds on Higgs boson mass

- perturbativity $\longrightarrow$ upper bound
- unitarity $\longrightarrow$ upper bound
- triviality (Landau pole) $\longrightarrow$ upper bound
- vacuum stability $\longrightarrow$ lower bound

perturbativity

decay widths into fermions: $\Gamma(H \to f\bar{f}) = \Gamma_{\text{tree}} \cdot K_f$

decay widths into vector bosons: $\Gamma(H \to V\bar{V}) = \Gamma_{\text{tree}} \cdot K_V$

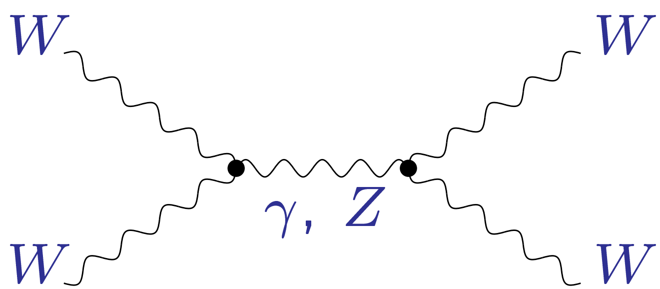


unitarity

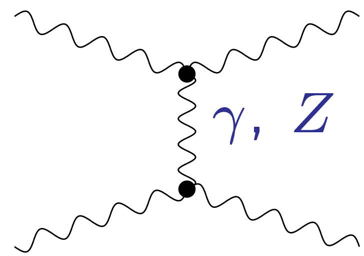
scattering of longitudinally polarized W bosons:

$$W_L W_L \to W_L W_L$$

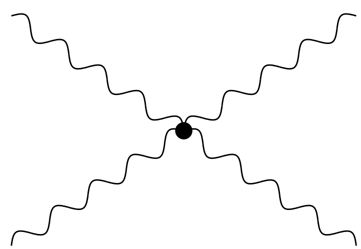
$\mathcal{M}_V =$



$+$

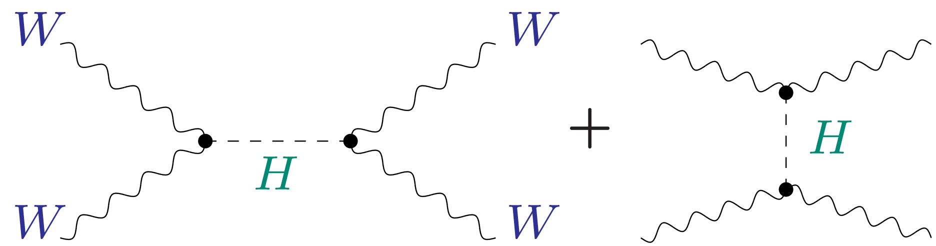


$+$



$$= -g^2 \frac{E^2}{M_{\mathrm{W}}^2} + \mathcal{O}(1) \qquad \text{for } E \to \infty$$

Extra contribution from scalar particle:

$\mathcal{M}_S =$ 

$$= g_{WWH}^2 \frac{E^2}{M_{\rm W}^4} + \mathcal{O}(1) \qquad \text{for } E \to \infty$$

$$\mathcal{M} \quad = \mathcal{M}_V + \mathcal{M}_S$$

$\Rightarrow$ terms with bad high-energy behavior cancel for

$$g_{WWH} = g\, M_{\rm W}$$

for $s >> M_W^2$, with $t = -\frac{s}{2}(1 - \cos\theta)$,

$$\mathcal{M} \approx \frac{M_H^2}{v^2}\left(2 + \frac{M_H^2}{s - M_H^2} + \frac{M_H^2}{t - M_H^2}\right)$$

partial wave expansion:

$$\mathcal{M}(s,t) \;=\; 8\pi \sum_{l=0}^{\infty} (2l+1)\, P_l(\cos\theta)\, a_l$$

unitarity condition: $|a_l| < 1$

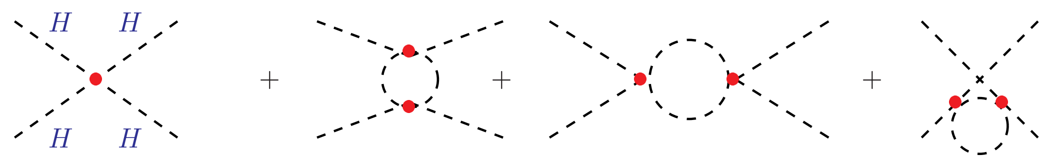
project on $l = 0$ partial wave:

$$\begin{aligned} a_0 &= \frac{1}{16\pi} \int_{-1}^{1} d\cos\theta\; \mathcal{M}(s,t) \\ &= \frac{M_H^2}{8\pi v^2} \left[2 + \frac{M_H^2}{s - M_H^2} - \frac{M_H^2}{s} \log\left(1 + \frac{s}{M_H^2}\right) \right] \\ &\approx \frac{M_H^2}{4\pi v^2} \quad \text{for} \quad s >> M_H^2 \end{aligned}$$

$$\boxed{a_0 < 1 \quad \Rightarrow \quad M_H < 872\,\mathrm{GeV}}$$

triviality (Landau pole)

Higgs self coupling is scale dependent, $\lambda(Q)$



variation with scale Q described by RGE

$$Q^2 \frac{d\lambda}{dQ^2} = \beta(\lambda) = \frac{3}{4\pi^2} \lambda^2$$

solution:

$$\lambda(Q) = \frac{\lambda(v)}{1 - \frac{3}{4\pi^2}\lambda(v) \log \frac{Q^2}{v^2}} \quad \text{with} \quad \lambda(v) = \frac{M_H^2}{2v^2}$$

diverges at scale $Q = \Lambda_C$ (Landau pole)

$$\Lambda_C = v \, \exp\left(\frac{4\pi^2 v^2}{3M_H^2}\right)$$

self-coupling diverges at

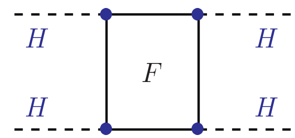
$$\Lambda_C = v \, \exp\left(\frac{4\pi^2 v^2}{3M_H^2}\right)$$

maximum Higgs mass by condition $\Lambda_C > M_H$

$$\Rightarrow \quad M_H < 800\,\mathrm{GeV}$$

vacuum stability

top-quark Yukawa coupling $g_t \sim m_t$ contributes to the running Higgs self coupling $\lambda(Q)$ through top loop $\quad \sim g_t^4$



variation with scale Q described by RGE

$$\frac{d\lambda}{dt} = \frac{3}{4\pi^2}\left(\lambda^2 - \frac{m_t^4}{v^4}\right)$$

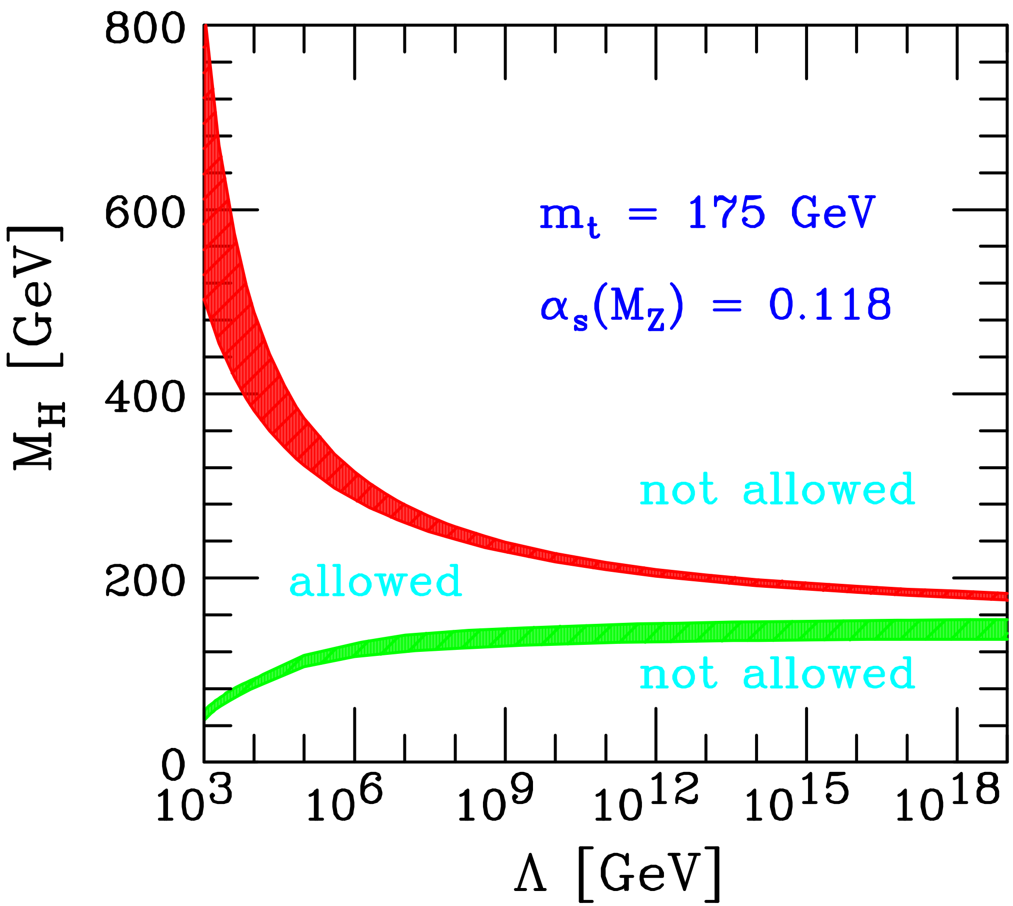
approximate solution:

$$\lambda(Q) = \lambda(v) - \frac{3m_t^4}{2\pi^2 v^4}\log\frac{Q}{v}$$

$\lambda(Q) < 0$ for $Q > \Lambda_C$ $\rightarrow$ vacuum not stable

high value of Λ_C needs M_H large enough combined effects, RGE in two-loop order:

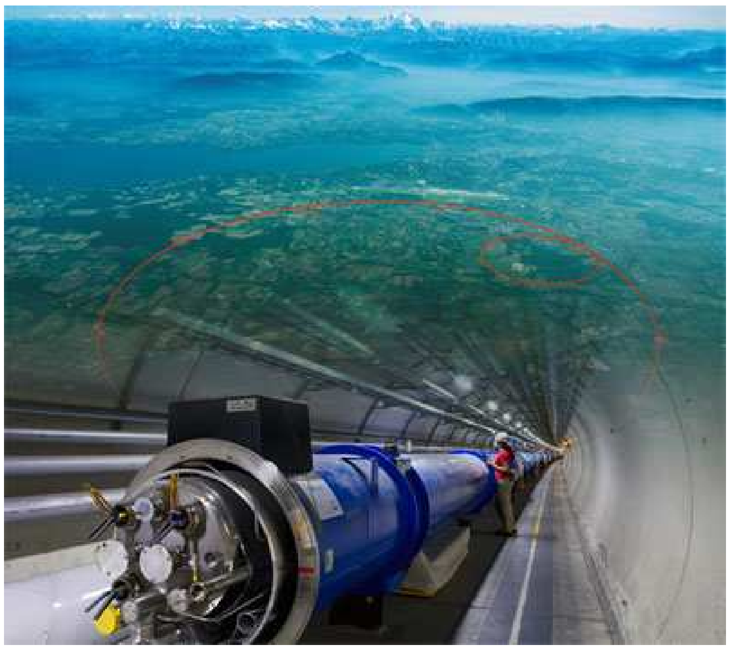
$$\frac{d\lambda}{dt} = \frac{1}{16\pi^2}\left(12\lambda^2 - 3\,g_t^4 + 6\,\lambda\,g_t^2\; + \cdots\right)$$



Perspectives

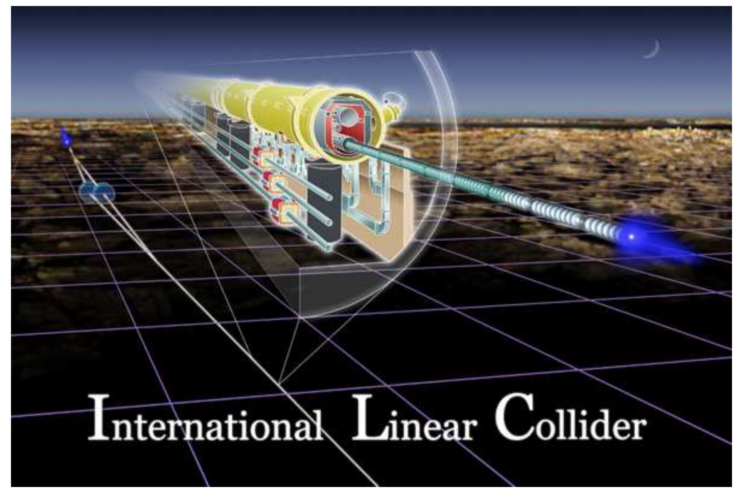
2009:

The Large Hadron Collider



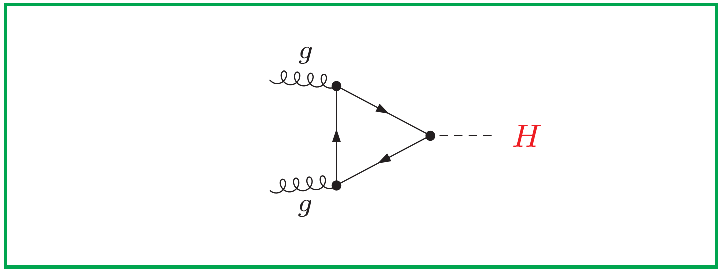
Future:

e^+e^- **Linear Collider**

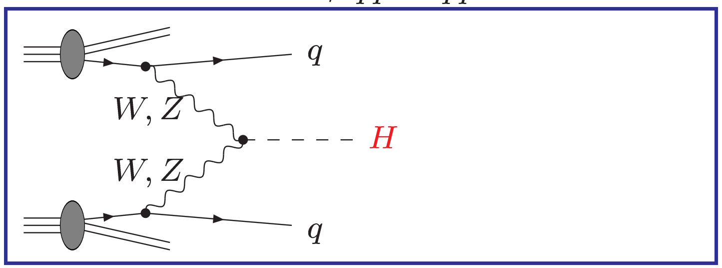


Higgs production at the LHC

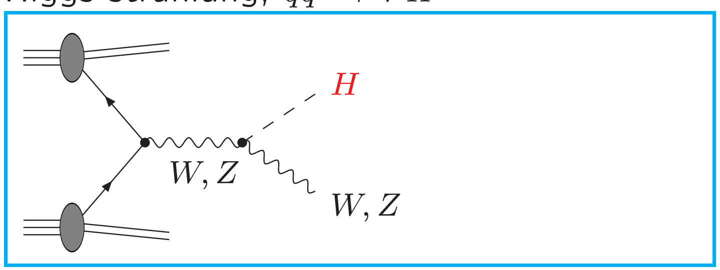
- gluon fusion, $gg \to H$



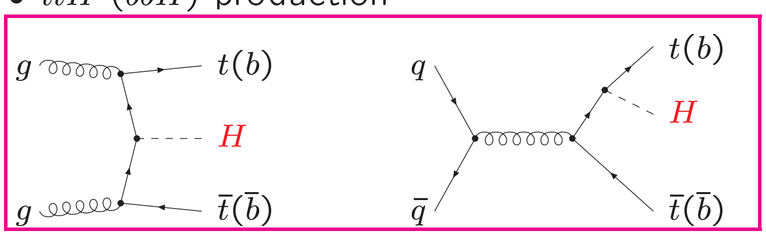
- vector boson fusion, $qq \to qqH$

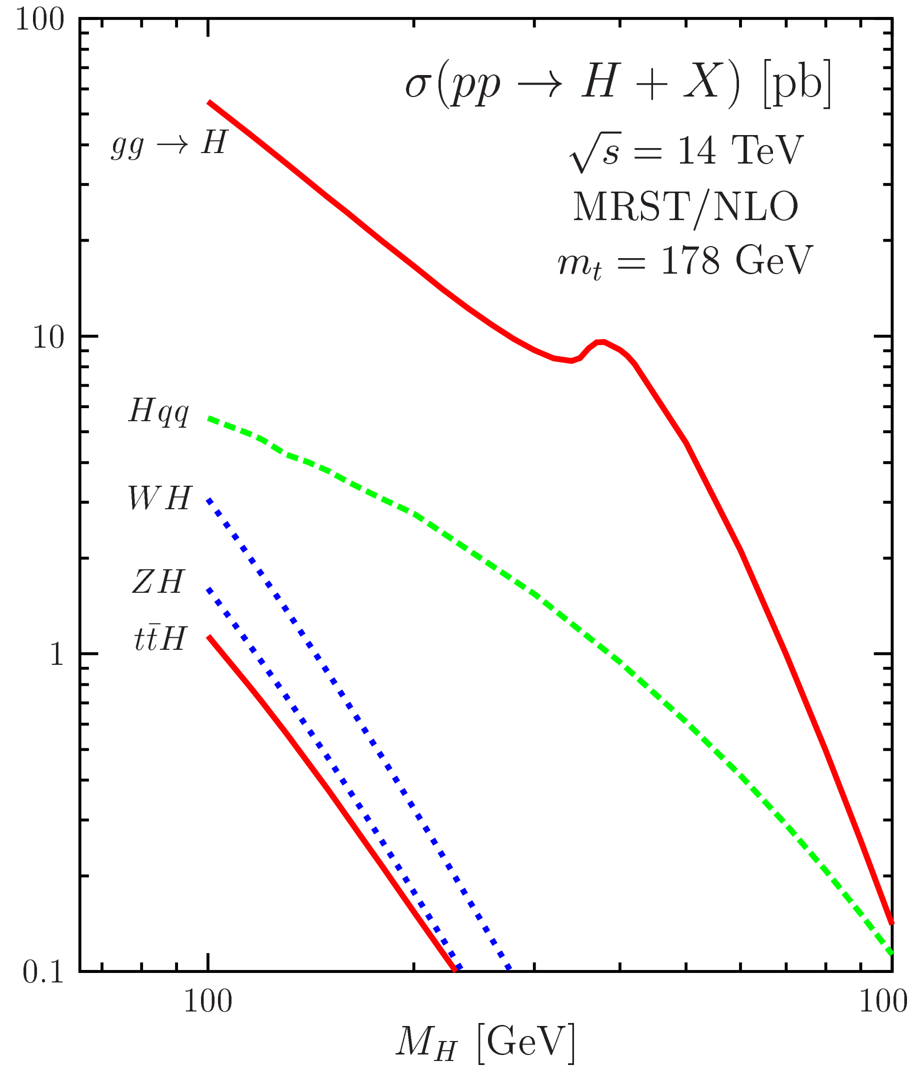


- Higgs strahlung, $q\bar{q}' \to VH$



- $t\bar{t}H$ ($b\bar{b}H$) production

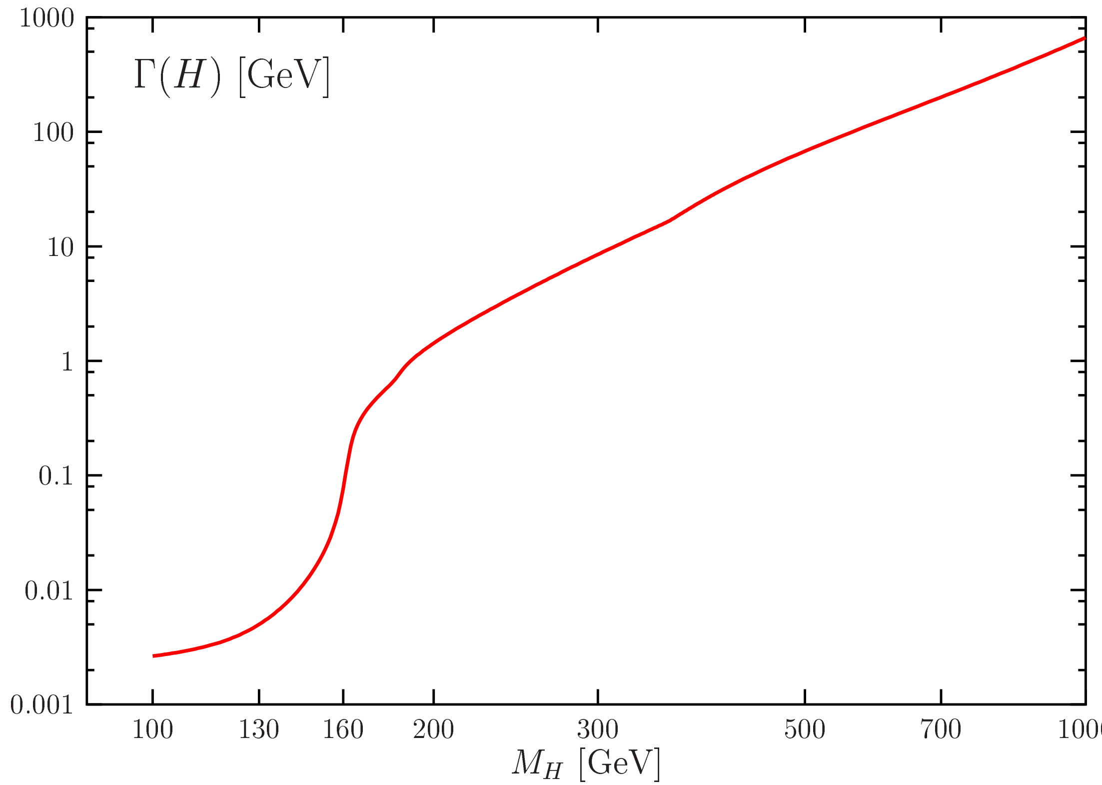




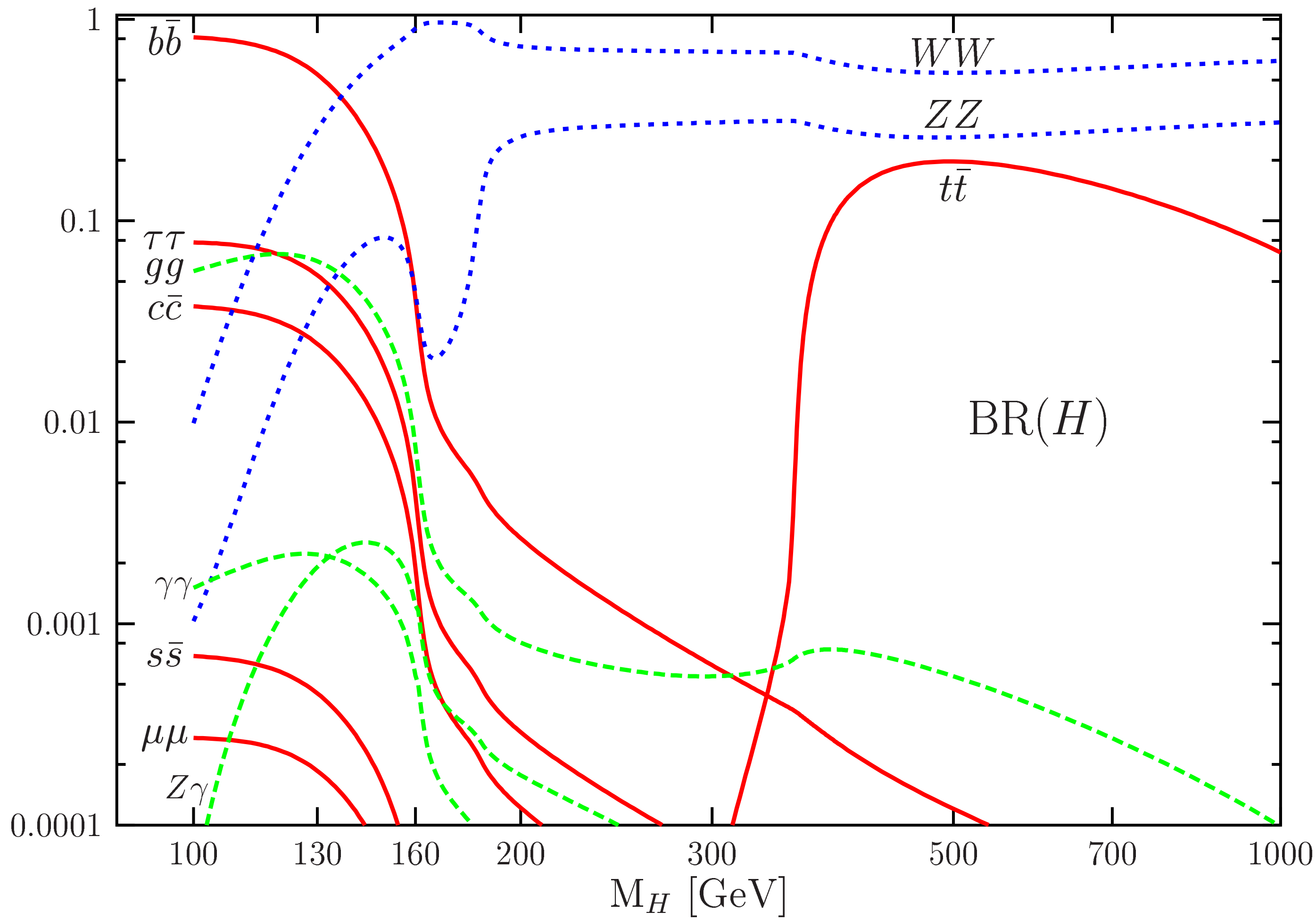
Higgs decays

- $H \to f\bar{f}$: $\Gamma \sim M_H\, m_f^2$
- $H \to WW,\, ZZ$: $\Gamma \sim M_H^3$

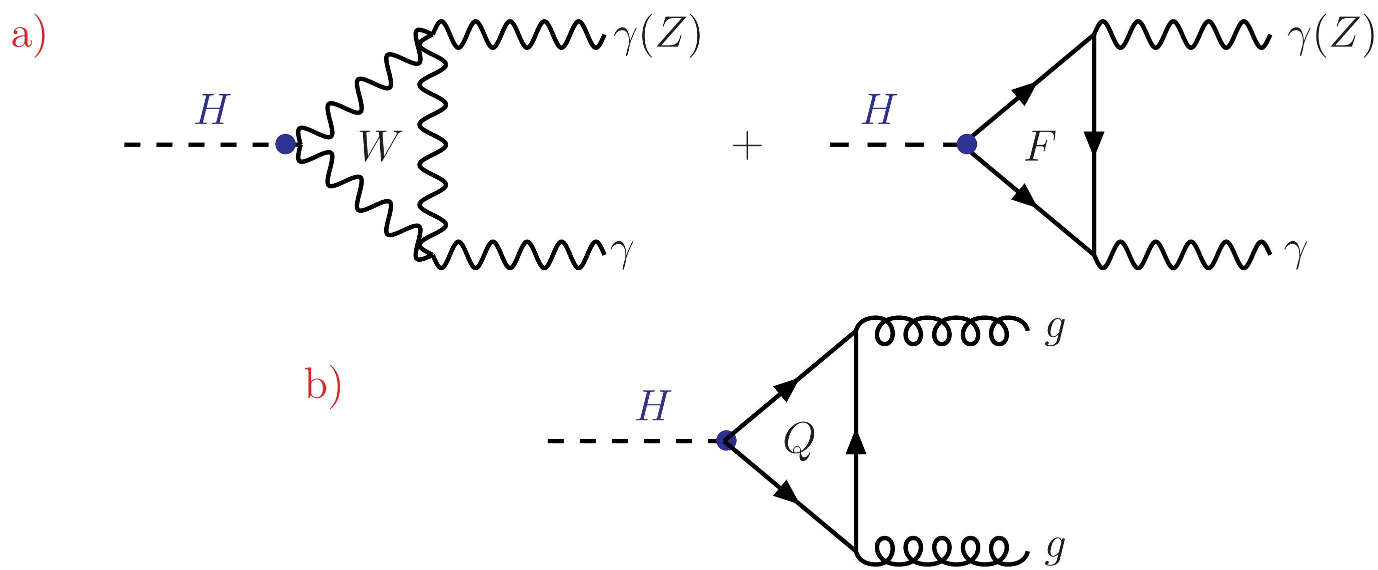
total width



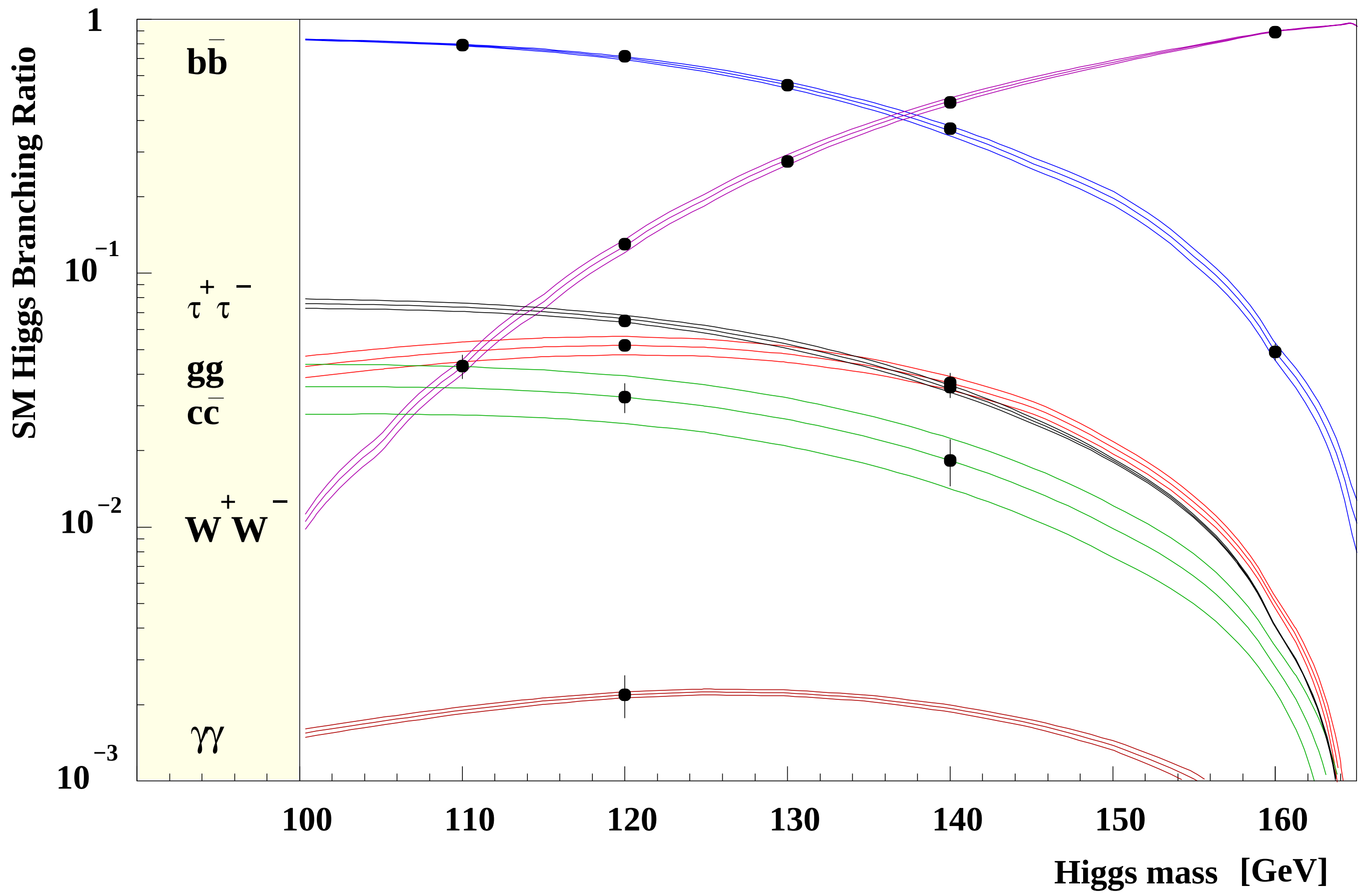
branching ratios



loop-induced decays



branching ratios at a Linear Collider



SM Higgs:

- $\lambda\, H^4$ term ad hoc
- Higgs boson mass: free parameter $\sim \sqrt{\lambda}$
- no a-priori reason for a light Higgs boson

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- λH^4 term ad hoc
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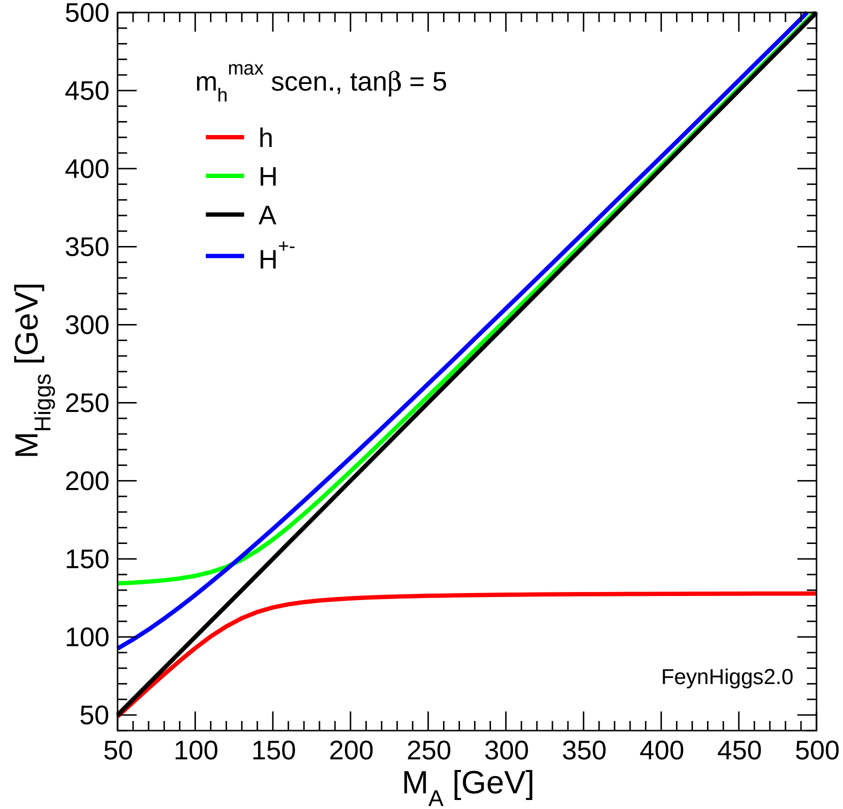
SUSY Standard Model avoids these questions

$$H_2 = \begin{pmatrix} H_2^+ \\ v_2 + H_2^0 \end{pmatrix}, \qquad H_1 = \begin{pmatrix} v_1 + H_1^0 \\ H_1^- \end{pmatrix}$$

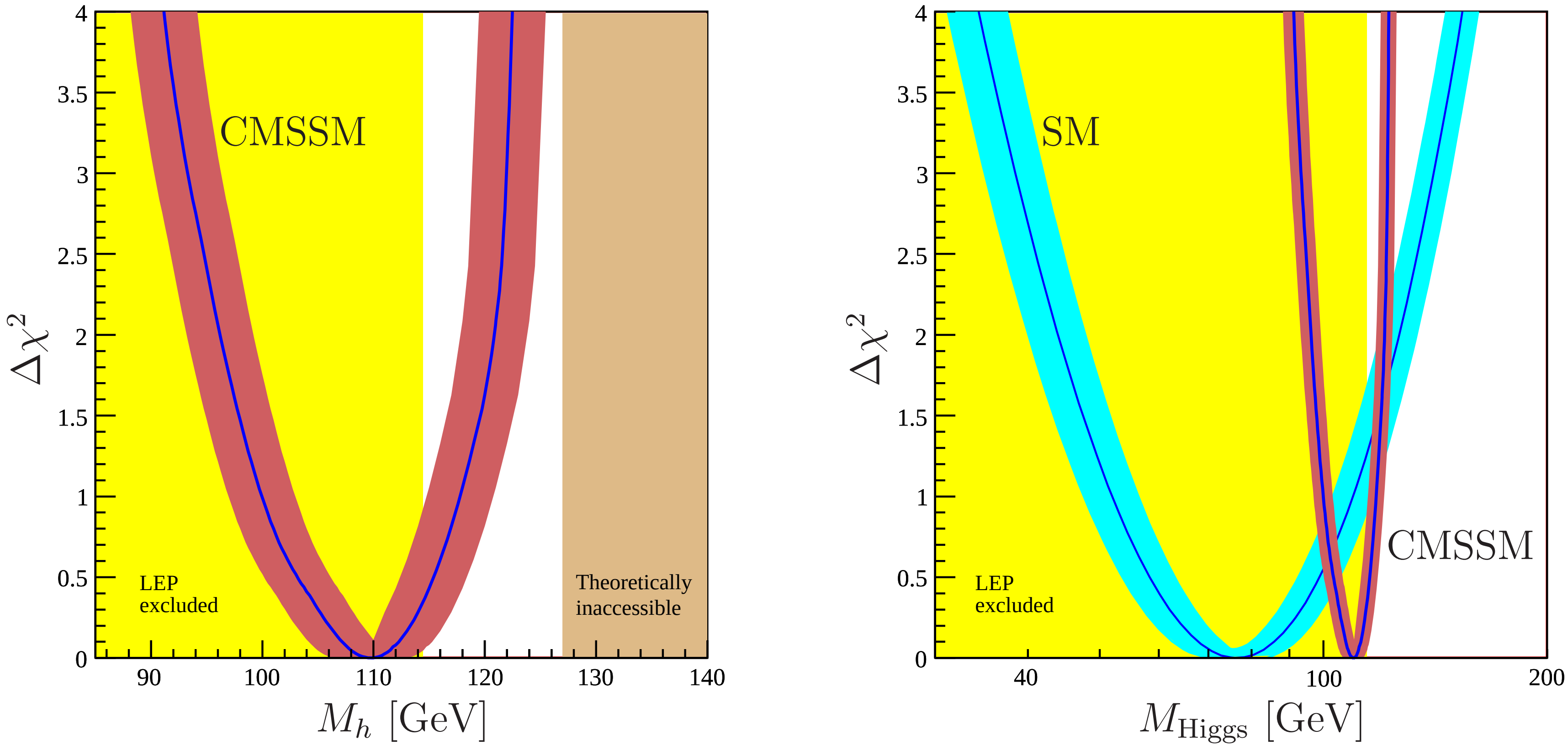
$$\text{couples to } u \qquad\qquad \text{couples to } d$$

- SUSY gauge interaction $\rightarrow$ H^4 terms
- self coupling remains weak

Spectrum of Higgs bosons in the MSSM (example)



large M_A: h^0 like SM Higgs boson $\sim$ decoupling regime



$$M_h = 110^{+8}_{-10}\ \text{GeV}$$