#### **Neutrino Physics**





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### **1. Introduction**

### The Birth of the Neutrino



W. Pauli

Before 1930: neutron → proton +e<sup>-</sup> 2-body decay → monoenergetic spectrum expected

<u>experiment: continuous β-decay spectrum</u> <u>Pauli: energy-momentum conservation</u>

- → postulate new particle
- $\rightarrow$  invisible, since Q=0
- → spin ½, ...
  Letter to Tübingen Dec. 1930 ...
  ... will never be detected

Cowen & Reines 1954-56 project ``poltergeist"
 → detection of reactor neutrinos
 → Nobel price for F. Reines 1995









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### **New Physics: Neutrino Sources**



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### **The Standard Model**

#### → success of renormalizable gauge field theories

 $\begin{array}{lll} \textbf{QED} \Rightarrow & \textbf{QCD} \Rightarrow & \textbf{SM} \\ \\ U(1)_{em} \Rightarrow & SU(3)_c \Rightarrow & SU(3)_c \times SU(2)_L \times U(1)_Y \end{array}$ 

- Singlet with respect to all symmetries
- Renormalizability
- Anomaly free combinations of chiral fermions

 $\label{eq:main_star} \text{Many details fixed by Lagrangian:} \quad \mathcal{L} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{fermion}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}}$ 

$$\mathcal{L}_{gauge} = -\frac{1}{2}Tr \left[G_{\mu\nu}G^{\mu\nu}\right] - \frac{1}{2}Tr \left[W_{\mu\nu}W^{\mu\nu}\right] - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \qquad \text{(adjoint representations)}$$
$$\mathcal{L}_{fermion} = \sum_{L} \overline{L} \ i\gamma^{\mu}D_{\mu}L + \sum_{r} \overline{r} \ i\gamma^{\mu}D_{\mu}r \qquad \text{(kinetic terms of all fermions)}$$
$$\mathcal{L}_{Higgs} = |D\Phi|^{2} - V(\Phi^{+}\Phi) \qquad \text{(Higgs potential} \quad \Leftrightarrow \quad \text{SSB)}$$

 $\mathcal{L}_{Yukawa} \simeq -g_Y \overline{L} \Phi r + h.c.$  (fermion masses, CKM-mixing, fermion-Higgs interaction) Manfred Lindner European School of High Energy Physics

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### **Problems of the Standard Model**

#### • The hierarchy problem

- how to stabilize  $m_H$  in an embedding?
- $\rightarrow$  SUSY, extra dimensions, composite, ...

### Strong CP problem

- why is CP-violation absent in strong sector
- $\rightarrow$  axions, ...

# Too many parameters in flavour sector → ? What are generations?

# **Physics Beyond the Standard Model**

#### **Theoretical arguments:**

SM does not exist without cutoff Higgs-doublett = only simplest extension Gauge hierarchy problem Why: 3 generations , fermion representations Many parameters (9+? Masses, 4+? Mixings) Charge quantisation, unification: GUTs, ..., Gravitation, ...

**<u>2 directions:</u>** Sym. breaking & Flavour

#### **Experimental facts:**

- Dark Matter & Dark Energy exist!
- Neutrino masses have been detected!
- **Baryon asymetry** of the universe  $\leftarrow \rightarrow m_{v} > 0$
- → physics beyond the standard model
- $\rightarrow$  results  $\leftarrow \rightarrow$  implications for theory



#### **Different Routes Beyond the SM**



# 2. Introducing Neutrino Masses & Mixings

### **Extending the Standard Model**

#### → success of renormalizable gauge field theories in d=4

QED 🗲	QCD	→ SM
<b>U(1)</b> <sub>em</sub>	<b>SU(3)</b> <sub>C</sub>	$SU(3)_C \times SU(2)_L \times U(1)_Y$

# symmetry, renormalizability, no anomalies particle content (symmetry representations):

gauge sector – fixed by gauge group scalar sector – must break EW symmetry, SB~2<sub>L</sub> fermions – anomaly free combinations

#### ➔ different levels of SM extension...

- add further SM representations
- extend the gauge symmetry
- add supersymmetry
- extend/modify basic concepts: quantum fields and/or space-time

### **Adding Neutrino Mass Terms**

#### 1) Postulate right handed neutrino fields -> SM+

Field	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
$ L_Q = \left(\begin{array}{c} l_u \\ l_d \end{array}\right) $	3	2	1/3
$r_u$	3	1	4/3
$r_d$	3	1	-2/3
$L_L = \begin{pmatrix} l_\nu \\ l_e \end{pmatrix}$	1	2	-1
$r_{\nu}$ ???	1	1	0
$r_e$	1	1	-2

not part of SM !
makes table more symmetric
3 right handed neutrinos?
<u>NEW:</u> → 9 parameters
→ explicit fermion mass term
→ L number violation

### **Adding Neutrino Mass Terms**

#### 1) Simplest possibility: add 3 right handed neutrino fields



#### NEW ingredients, 9 parameters -> SM+



### **Suggestive Seesaw Features**

**QFT: natural value of mass operators ← → scale of symmetry** 

 $m_D \sim$  electro-weak scale

 $M_R \sim L$  violation scale  $\Leftarrow$ ?  $\Rightarrow$  embedding (GUTs, ...)



Numerical hints:

For  $m_3 \sim (\Delta m_{atm}^2)^{1/2}$ ,  $m_D \sim leptons \Rightarrow M_R \sim 10^{11} - 10^{16} \text{GeV}$  $\Rightarrow v$ 's are Majorana particles,  $m_v$  probes  $\sim \text{GUT scale physics!}$  $\Rightarrow \text{smallness of } m_v \Leftarrow \Rightarrow \text{ high scale of } I/, \text{ symmetries of } m_D, M_R$ 

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## **2nd Look Questions**

Quarks & charged leptons → hierarchical masses → neutrinos?



- less hierarchy in  $m_D$  or correlated hierarchy in  $M_R$ ?  $\rightarrow$  theoretically connected!
- mixing patterns: not generically large, why almost maximal,  $\theta_{13}$  small?

### **Other effective Operators Beyond the SM**

#### → effects beyond 3 flavours

→ Non Standard Interactions = NSIs → effective 4f opersators

$$\mathcal{L}_{NSI} \simeq \epsilon_{lphaeta} 2\sqrt{2}G_F(\bar{
u}_{Leta} \ \gamma^{
ho} \ 
u_{Llpha})(\bar{f}_L\gamma_{
ho}f_L)$$

• integrating out heavy physics (c.f.  $G_F \leftarrow \Rightarrow M_W$ )



Grossman, Bergmann+Grossman, Ota+Sato, Honda et al., Friedland+Lunardini, Blennlow+Ohlsson+Skrotzki, Huber+Valle, Huber+Schwetz+Valle, Campanelli +Romanino, Bueno et al., Barranco+Miranda+Rashba, Kopp+ML+Ota, ...

## **Parameters for 3 Light Neutrinos**

mass & mixing parameters:  $m_1$ ,  $\Delta m_{21}^2$ ,  $|\Delta m_{31}^2|$ , sign( $\Delta m_{31}^2$ )



### **Overview of Neutrino Mass Determinations**



#### **3. Neutrino Mass Determinations**

### Four Methods of Mass Determination

- kinematical
- lepton number violation
   ←→ Majorana nature
- astrophysics & cosmology
- oscillations

#### **Kinematical Mass Determination**

model independent neutrino mass from ß-decay kinematics

$$\frac{\mathrm{d}\Gamma_i}{\mathrm{d}E} = C p \left(E + m_e\right) \left(E_0 - E\right) \sqrt{\left(E_0 - E\right)^2 - m_i^2} F(E) \theta \left(E_0 - E - m_i\right)$$

$$C = G_F^2 \frac{m_e^5}{2 \pi^3} \cos^2 \theta_C \, |M|^2$$



experimental observable: m<sub>v</sub><sup>2</sup>

#### **ß-source requirements :**

- high  $\beta$ -decay rate (short  $t_{1/2}$ )
- low β-endpoint energy E<sub>0</sub>
- superallowed ß-transition
- minimised inelastic scatters of B's

#### **B-detection requirements :**

- high energy resolution (< few eV)
- large solid angle ( $\Delta \Omega \sim 2\pi$ )
- low background

### **Status of kinematical Mass Determination**



#### Troitsk

windowless gaseous T<sub>2</sub> source

analysis 1994 to 1999, 2001

 $m_v^2$  = -2.3 ± 2.5 ± 2.0 eV<sup>2</sup>  $m_v \le 2.2$  eV (95% CL.)

#### Mainz

quench condensed solid T<sub>2</sub> source

analysis 1998/99, 2001/02

 $\label{eq:m_v} \begin{array}{l} m_{\nu}^2 = - \ 1.2 \pm 2.2 \pm 2.1 \ eV^2 \\ m_{\nu} \leq \quad 2.2 \ eV \ (95\% \ CL.) \end{array}$ 

both experiments have reached their intrinsic sensitivity limit

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#### **The Future**

#### **KATRIN - Karlsruhe Tritium Neutrino Experiment**

#### direct n-mass measurement with sub-eV sensitivity



#### **Kinematical Mass Determination** 1.2 100 a) **Relativistic kinematics:** b) count rate [a.u.] 9.0 8 80 count rate [a.u.] $E^{2} = p^{2} + m^{2}; \ \sum p_{i}^{\mu} = \sum p_{f}^{\mu}$ 60 $m_v = 0 eV$ 40 Endpoint of decays: 2 x 10<sup>-13</sup> 20 Tritium $\rightarrow He^3 + e^- + \overline{\nu}_e$ 0.2 $m_v = 1 \text{ eV}$ 0 0 15 $\cap$ 5 10 -2-0--2 \_ 1 0 -3 E-En [eV]

	"Elektron-Neutrino" :	$m < 2.2 \ { m eV}$	(Mainz, Troitsk)
Bounds:	"Muon-Neutrino" :	m < 170  keV	
	"Tau-Neutrino":	m < 15.5 MeV	

energy E [keV]

Sensitivity 
$$\Leftrightarrow$$
 degenerate  $\nu$ -spectrum  
 $\Rightarrow$  Oscillations:  $\Delta m_{ij}^2 \ll m_i^2 \Rightarrow \qquad \sum m_i^2 |U_{ei}|^2 < (2.2 \text{ eV})^2$ 

Future: KATRIN → 0.20 eV

#### $\leftarrow$ $\rightarrow$ c.f. comological bounds

### Four Methods of Mass Determination

- kinematical
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- astrophysics & cosmology
- oscillations

#### **Double Beta Decay**



# → 2 neutrinos plus 2 electrons



#### **Double Beta Decay: Mass Parabolas**



### **0νββ Decay Kinematics**



#### Majorana ν **→** 0νββ decay

#### warning:

other lepton number violating processes...

2νββ decay of <sup>76</sup>Ge observed:  $\tau = 1.5 \times 10^{21}$  y



- signal at known Q-value
- 2vββ background (resulution)
- nuclear backgrounds
  - ➔ use different nuclei

#### **Relating Rates / Lifetimes to Neutrino Masses**



nuclear matrix elements:

→ virtual excitations of intermediate states

Fäßler et al., ...







**2ν2**β:

$$\begin{bmatrix} T_{\frac{1}{2}}^{2\nu} (0^{+} \rightarrow 0^{+}) \end{bmatrix}^{-1} = G^{2\nu}(E_{0},Z) M_{GT}^{2\nu} - \frac{g_{V}^{2}}{g_{A}^{2}} M_{F}^{2\nu} \end{bmatrix}^{2}$$
Phase space nuclear matrix element
(assuming that leading term is due to exchange of light Majorana-neutrino)
$$\begin{bmatrix} T_{\frac{1}{2}}^{0\nu}(0^{+} \rightarrow 0^{+}) \end{bmatrix}^{-1} = G^{0\nu}(E_{0},Z) M_{GT}^{0\nu} - \frac{g_{V}^{2}}{g_{A}^{2}} M_{F}^{0\nu} \qquad (m_{V})^{2}$$
Remark: 0v2\beta also generated by SUSY, LR ....

### **Nuclear Matrix Elements**

**0**v**2**β half-lives in units of 10<sup>26</sup> years for  $< m_v > = 50$  meV for nuclear matrix of different authors

**Attention: systematically** correleated calculatio

cally ons!	Nat	net di.	stet al.	et al.	dtet al. aess	eret al.	set al.
Nucleus	Ref.: (20)	(80)	(81)	(82)	(24,83)	(84)	:
$^{48}Ca$	12.7	35.3	-	-	-	10.0	
$^{76}\mathrm{Ge}$	6.8	70.8	56.0	9.3	12.8	14.4	
$^{82}$ Se	2.3	9.6	22.4	2.4	3.2	6.0	
$^{100}\mathrm{Mo}$	-	-	4.0	5.1	1.2	15.6	
$^{116}\mathrm{Cd}$	-	-	-	1.9	3.1	18.8	
$^{130}\mathrm{Te}$	0.6	23.2	2.8	2.0	3.6	3.4	
$^{136}\mathrm{Xe}$	-	48.4	13.2	8.8	21.2	7.2	
$^{150}\mathrm{Nd}^a)$	-	-	-	0.1	0.2	-	
$^{160}\mathrm{Gd}^a)$	-	-	-	3.4	-	-	

#### How to measure $0\nu 2\beta$ decay ?



#### Heidelberg-Moscow Experiment @ LNGS

Detector number	Total mass (kg)	Active mass (kg)	Enrichment in <sup>76</sup> Ge(%)	PSA
No. 1	0.980	0.920	$85.9 \pm 1.3$	No
No. 2	2.906	2.758	$86.6 \pm 2.5$	Yes
No. 3	2.446	2.324	$88.3 \pm 2.6$	Yes
No. 4	2.400	2.295	$86.3 \pm 1.3$	Yes
No. 5	2.781	2.666	$85.6 \pm 1.3$	Yes

Technical parameters of the five enriched <sup>76</sup>Ge detectors





Fig. 1. The HEIDELBERG–MOSCOW  $\beta\beta$ -experiment in the Gran Sasso (top), and four of the enriched detectors during installation (bottom left). The fifth detector was installed in an extra shielding using electrolytic copper as inner shield (bottom right).

Data acquisition and analysis of the <sup>76</sup>Ge double beta experiment in Gran Sasso 1990–2003

NIM A 522 (2004)

H.V. Klapdor-Kleingrothaus<sup>\*,1</sup>, A. Dietz, I.V. Krivosheina<sup>2</sup>, O. Chkvorets

#### **Evidence by Part of HM**



Fig. 17. The total sum spectrum of all five detectors (in total 10.96 kg enriched in <sup>76</sup>Ge), for the period November 1990–May 2003 (71.7 kg year) in the range 2000–2060 keV and its fit (see Section 3.2).

Nov 1990- May 2003
71.7 kg year
Bgd 0.11 / kg y keV
28.75 ± 6.87 events (bgd:~60)
4.2 sigma evidence for 0vββ
0.34-2.03 x10<sup>25</sup> y (3 sigma)
Best fit 1.19 x10<sup>25</sup> y

•m<sub>ee</sub> = 0.1-0.9 eV •best fit 0.44 eV
#### **SSE\*** Analysis: Strengthening the Evidence?



## **Other running Experiments**

**CUORICINO (Cryogenic Underground Observatory for Rare Events):** Firenze, Gran Sasso, Insubria, LBNL, Leiden, Milano, Neuchatel, South Carolina, Zaragoza

Location: Gran Sasso Underground Laboratory Source = detector, TeO<sub>2</sub> (40 kg)  $\Rightarrow$  <sup>130</sup>Te (13 kg): Q = 2533±4 keV

## $\tau_{1/2} > 1.8 \ 10^{24}$ at 90% C.L. (<mv> < [0.2+1.1] eV)

#### **NEMO3** (Neutrino Ettore Majorana Observatory):

CENBG Bordeaux, Charles Univ. Prague, FNSPE Prague, INEEL, IReS Strasbourg, ITEP Moscow, JINR Dubna, Jyvaskyla Univ., LAL Orsay, LPC Caen, LSCE Gif, Mount Holyoke College, Saga Univ, UCL London

Location: Frejus Underground Laboratory

Source  $\neq$  detector  $\Rightarrow$  study different nuclei; main target <sup>100</sup>Mo (6.9 kg): Q = 3034\pm6keV

#### Results → so far mass limits weaker as HM ; O(0.7-5eV)

## **New Experiments**

#### **Advanced construcion:**

- CUORE (Te-130) → 2010?
- GERDA (Ge-76) **→** start 2009!

#### **Under construction:**

- Majorana
- EXO
- MOON
- Super-NEMO
- ...

## **GERDA** Construction





Vacuum-insulated double wall stainless steel cryostat



#### → data taking 2009

### **Neutrino-less Double β-Decay**





#### aims of new experiments:

- test HM claim
- (∆m<sub>31</sub><sup>2</sup>)<sup>1/2</sup> ~ 0.05eV ± errors
   → reach 0.01eV
  - → CUORE
  - → GERDA phases I, II, (III)



#### **Comments:**

- cosmology: limitation by systematical errors → ~another factor 5?
- $0\nu\beta\beta$  nuclear matrix elements ~factor 1.3-2 theoretical uncertainty in m<sub>ee</sub>
- $\Delta m^2 > 0$  allows complete cancellation  $\rightarrow 0\nu\beta\beta$  signal not guaranteed
- $0\nu\beta\beta$  signal from \*some other\* new BSM lepton number violating operator
  - very promising interplay of neutrino mass determinations, cosmology, LHC, LVF experiments and theory

#### <u>alternatives:</u> LR, RPV-SUSY, ... → other *L* operators ← → NSI's



#### **Schechter+Valle:**

L violating operator  $\rightarrow$  radiative mass generation  $\rightarrow$  Majorana nature of v's However: This may only be a tiny correction to a much larger Dirac mass term

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## **Lepton Flavour Violation**

- Majorana neutrino mass terms
- **R-parity violating supersymmetry** Hall+Kosteleck+Rabi, Borzumati+Masiero, Hisano+Tobe, Casas+Ibarra, Antusch +Arganda+Herrero+Teixeira, Joaquim+Rossi, ...



 $\rightarrow$  interplay: v's – LFV - LHC



## Four Methods of Mass Determination

- kinematical
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   ←→ Majorana nature
- astrophysics & cosmology
- oscillations

## **The thermal evolution of the Universe:**

## 10<sup>-43</sup> seconds

speculative physics: 10<sup>19</sup>GeV:Strings, ...

Time





## 10<sup>-34</sup> seconds



# 10<sup>27</sup>degrees

GUT physics: 10<sup>16</sup>GeV







# 10<sup>9</sup> degrees Synthesis of light elements

# 300 thousand He vears Universe becomes transparent → 3K radiatio 6000 degrees

# 1 thousand million years



Structure formation, molecules, ...

**18 degrees** 

# Today: 15 thousand million years

 $T_{\gamma} \simeq 2.7 \text{K}, T_{\nu} \simeq 1.7 \text{K}$ BBN works for N<sub>\nu</sub>=3 330 Neutrinos / cm3 Mass: Neutrinos  $\leq$  baryons

## **Neutrinos & Cosmology**

- Dark Matter ~ 25% & Dark Energy 70%
- mass of all neutrinos:  $0.001 \le \Omega_v \le 0.02$
- baryonic matter  $\Omega_{\rm B} \sim 0.04$

Neutrino mass contribution possibly as big as all baryonic matter >> visible matter much more COLD dark matter & dark energy neutrinos are an important hot dark matter component

**Present Day Acceleration** 

## **Comological impact of neutrinos:**

- hot component in structure formation: 330v/cm<sup>3</sup> x mass -

- Big Bang Nuklueosynthesis  $\rightarrow$
- Baryon asymmetry  $\rightarrow$  Leptogenesis  $\rightarrow$  ?

-...

## **Cosmology and Neutrino Mass**



## **Baryon Asymmetry & Neutrinos**



measured baryon asymmetry:  $\eta = \frac{n_B}{n_{\gamma}} = 4(3) \cdot 10^{-10} \dots 7(10) \cdot 10^{-10}$ sphalerons **Necessary: Sakharov conditions:** R• B-violating processes ←→ sphalerons • C- and CP-violation  $\leftarrow \rightarrow$  contained in model chemical potential • departure from thermal equilibrium  $\leftarrow \rightarrow \Gamma < H$ analysis  $(\mathbf{S})$ natural explanation of leptogenesis baryon asymmetry by  $\Delta L$  $(\mathbf{d})$  minimal leptogenesis works nicely • different interesting variants ... a talk by itself

## Four Methods of Mass Determination

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## **Two Neutrino Oscillations**

**2** Neutrinos:  $v_e, v_\mu$ 

 $\begin{aligned} |\nu_e(0)\rangle &= \cos\theta \, |\nu_1\rangle + \sin\theta \, |\nu_2\rangle \\ |\nu_\mu(0)\rangle &= -\sin\theta \, |\nu_1\rangle + \cos\theta \, |\nu_2\rangle \end{aligned}$ 



$$|\nu_{\mu}(t)\rangle = -\sin\theta \exp[-\frac{iE_{1}t}{\hbar}] |\nu_{1}\rangle + \cos\theta \exp[-\frac{iE_{2}t}{\hbar}] |\nu_{2}\rangle$$

$$E_i = \sqrt{p_i^2 + m_i^2} \xrightarrow{p_i = p \gg m_i} \simeq p + \frac{m_i^2}{2p} \simeq p + \frac{m_i^2}{2E}$$
$$L = c \cdot t \qquad \Delta m^2 = m_2^2 - m_1^2 \Rightarrow \quad E_2 - E_1 = \frac{\Delta m^2}{2E}$$

2v-transitionprobability:

$$P(\nu_{\mu} \to \nu_{e}) = \left| \langle \nu_{\mu}(t) | \nu_{e}(0) \rangle \right|^{2} = \sin^{2} 2\theta \cdot \sin^{2} \left( \frac{\Delta m^{2} L}{4E} \right)$$

$$v_e, v_\mu, v_\tau \rightarrow 9$$
 oscillation channels for neutrinos  
 $v_e, v_\mu, v_\tau \rightarrow 9$  channels for anti-neutrinos (assuming  $3v$  !)

## **Oscillations in QFT**

- is ordinary QM sufficient to describe v-oscillations?
- v's are relativistic, 2nd quantization, ...
  - → Feynman diagram of neutrino oscillation:
    - energy momentum properties, quantum numbers
    - → QM limit, coherence, kinematics, ...
    - e.g. observation of solar neutrinos in  $v_{e}$  channel



### **Neutrino Oscillations in QFT**

## QFT description of a neutrino produced in a decay at rest:

- localized source and detector
- $L = |\vec{x}_D \vec{x}_S|$
- initial particle at rest
- target particle at rest

#### ... DIF similar



#### **Transition probability from Feynman diagram:**

$$\left\langle P_{\substack{(-)\\\nu_{\alpha}\rightarrow}}^{(-)} \right\rangle_{\mathcal{P}} \propto \int dP_S \int_{\mathcal{P}} \frac{d^3 p_{D1}}{2E_{D1}} \cdots \frac{d^3 p_{Dn_D}}{2E_{Dn_D}} \left| \mathcal{A}_{\substack{(-)\\\nu_{\alpha}\rightarrow}}^{(-)} \right|^2$$

#### $\Rightarrow$ leads to neutrino oscillation + avoids confusion ...

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#### **Kinematics: Equal Energy or equal Momenta?**

- Consider e.g. pion decay at rest:  $\pi^+ \rightarrow \mu^+ + \nu_{\mu}$
- Neutrino energy and momentum determined by energy-momentum conservation

$$p_k^2 = \frac{m_\pi^2}{4} \left( 1 - \frac{m_\mu^2}{m_\pi^2} \right)^2 - \frac{m_k^2}{2} \left( 1 + \frac{m_\mu^2}{m_\pi^2} \right) + \frac{m_k^4}{4 m_\pi^2}$$
$$E_k^2 = \frac{m_\pi^2}{4} \left( 1 - \frac{m_\mu^2}{m_\pi^2} \right)^2 + \frac{m_k^2}{2} \left( 1 - \frac{m_\mu^2}{m_\pi^2} \right) + \frac{m_k^4}{4 m_\pi^2}$$

• For 
$$E \gg m$$
:  $p_k \simeq E - \xi \frac{m_k^2}{2E}$ ,  $E_k \simeq E + (1 - \xi) \frac{m_k^2}{2E}$   
with  $E = \frac{m_\pi}{2} \left( 1 - \frac{m_\mu^2}{m_\pi^2} \right) \simeq 30 \,\text{MeV}$ ,  $\xi = \frac{1}{2} \left( 1 + \frac{m_\mu^2}{m_\pi^2} \right) \simeq 0.8$ 

⇒ neither equal energy nor equal momentum!

$$e^{ipx} \Rightarrow \left[ p_{\mu} \cdot x^{\mu} = p_k L - E_k T = -\frac{m_k^2 L}{2E} \right]$$
 for  $L = T$ 

 $\Rightarrow \xi$  drops out of the oscillation formulae  $\Leftrightarrow$  naive treatment correct

• Shown for  $\pi$ -decay, but valid in general (DIF, N-body, ..., different  $\xi$ )

#### **Localized Source and Detector:**

- Feynman rules for particles of given momentum ( $\simeq$  on-shell)
  - $\Rightarrow$  this corresponds to an infinitely extended (non-localized) plane wave
- Localized source (wave packet) and detector in space-time ( $\Delta x_S, \Delta t_S$ ), ( $\Delta x_D, \Delta t_D$ ):
  - $\Rightarrow$  Source: Fourier superposition of momenta with  $\sigma_S^2 \simeq min(\Delta x_S^2, \Delta t_S^2)$
  - $\Rightarrow$  Detector: projection on a superposition of momenta with  $\sigma_D^2 \simeq min(\Delta x_D^2, \Delta t_D^2)$
- Different masses and momenta  $\Rightarrow$  dispersion  $\Rightarrow$  loss of coherence



- Oscillations from QFT  $\Rightarrow P_{\nu_{\alpha} \to \nu_{\beta}}(L,T) = \left|\sum_{k} U_{\alpha k}^{*} e^{ip_{k}L iE_{k}T} U_{\beta k}\right|^{2}$
- Very interesting QM effects ( $\sigma$ , decay)

#### **General 3x3 neutrino mixing matrix:**

has (up to) 3 angles + 1 Dirac-phase +2 Majorana-phases:  $\theta_{12}$ ,  $\theta_{23}$ ,  $\theta_{13}$ ,  $\delta$ ,  $\Phi_1$ ,  $\Phi_2$ 

$$U_{MNS} = U \cdot \operatorname{diag}(\exp[\mathbf{i}\Phi_1], \exp[\mathbf{i}\Phi_2], \mathbf{1})$$

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

• Only U enters in neutrino oscillations:  $\int J_{ij}^{e_l e_m} := U_{li} U_{lj}^* U_{mi}^* U_{mj}$ 

• All oscillation frequencies show up: 
$$\Delta_{ij} := \frac{\Delta m_{ij}^2 L}{4E} = \frac{(m_i^2 - m_j^2)L}{4E}$$

$$P(\nu_{e_l} \to \nu_{e_m}) = \underbrace{\delta_{lm} - 4 \sum_{i>j} \operatorname{Re} J_{ij}^{e_l e_m} \sin^2 \Delta_{ij}}_{P_{CP}} \underbrace{-2 \sum_{i>j} \operatorname{Im} J_{ij}^{e_l e_m} \sin 2\Delta_{ij}}_{P_{CP}}$$

⇒ Leptonic CP violation, genuine 3 flavour and matter effects

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Neutrinos: $P(\nu_{e_l} \rightarrow \nu_{e_m}) = P_{CP} + P_{CP}$ Antineutrinos: $P(\overline{\nu}_{e_l} \rightarrow \overline{\nu}_{e_m}) = P_{CP} - P_{CP}$ 

#### $\Rightarrow$ projecting out CP Asymmetries:

$$\mathbf{a^{CP}} := \frac{\mathbf{P}(\nu_{\mathbf{e_l}} \to \nu_{\mathbf{e_m}}) - \mathbf{P}(\overline{\nu}_{\mathbf{e_l}} \to \overline{\nu}_{\mathbf{e_m}})}{\mathbf{P}(\nu_{\mathbf{e_l}} \to \nu_{\mathbf{e_m}}) + \mathbf{P}(\overline{\nu}_{\mathbf{e_l}} \to \overline{\nu}_{\mathbf{e_m}})} = \frac{P_{CP}}{P_{CP}}$$

#### ... in practise not very useful:

- $\bullet$  different systemytics for  $\nu$  and  $\overline{\nu}$
- different statistics
- only one type of neutrinos

#### $\Rightarrow$ global fits with masses, mixings and CP parameters

#### **Matter Effects and MSW Resonance**

Mikheyev-Smirnov-Wolfenstein: coherent forward scattering



 $\mathcal{L}_{NC} = \mathsf{flavour} \ \mathsf{universal}$  $\mathcal{L}_{CC} = \sqrt{2}G_F n_e \quad \Leftrightarrow \quad \mathsf{only} \ \nu_e$ 

MSW-resonance energy( $\Delta m_{31}^2$ ) Earth: E<sub>res</sub>  $\simeq$  10 GeV

for beams dominated by average density

 $\rho = \rho_{\rm average} + \delta \rho$ 

## **Baseline & MSW Matter Effect**



•  $E_{resonance} \simeq 10 - 15$  GeV, matter effects grow with distance L

 $\bullet$  Average density profile uncertainties decrease with L  $\Rightarrow$   $~\simeq 5\%$  error

#### Hamiltonian for 3 Neutrino Oscillations in Flavour Basis:

$$\mathbf{H} = H_0 + \delta \mathbf{H}_{CC} + \delta \mathbf{H}_{NC} = \frac{1}{2E} \mathbf{U} \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix} \mathbf{U}^{\mathrm{T}} + \frac{1}{2E} \begin{pmatrix} \mathbf{A} + \mathbf{A}' & 0 & 0 \\ 0 & \mathbf{A}' & 0 \\ 0 & 0 & \mathbf{A}' \end{pmatrix}$$

• 
$$\mathbf{A} = \pm \frac{2\sqrt{2}\mathbf{G}_{\mathbf{F}}\mathbf{Y}\rho\mathbf{E}}{\mathbf{m}_{\mathbf{n}}} = 2V \cdot E$$
  $\nu \oplus \text{matter}$  and  $\overline{\nu} \oplus \text{anti} - \text{matter} \Rightarrow "+"$ 

•  $Y = e^{-}$ /nucleon  $\rho$  =matter density  $m_n$  =nucleon mass

• Overall phases drop out:  $m_i 
ightarrow m_i - m_1 \Rightarrow m_1$  and A' can be eliminated

$$\mathbf{H'} = \frac{1}{2E} \mathbf{U} \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m_{21}^2 & 0 \\ 0 & 0 & \Delta m_{31}^2 \end{pmatrix} \mathbf{U^T} + \frac{1}{2E} \begin{pmatrix} \mathbf{A} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

**Manfred Lindner** 

- In good approximation  $\Delta m^2_{12} \simeq 0$
- U can be written as a sequence of rotations:  $U = R_{23}R_{13}R_{12}$

$$\begin{split} \mathbf{H}^{\prime\prime} &= \frac{1}{2E} \mathbf{R_{23}} \mathbf{R_{13}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \Delta m_{31}^2 \end{pmatrix} \mathbf{R_{13}^{-1}} \mathbf{R_{23}^{-1}} + \frac{1}{2E} \mathbf{R_{23}} \begin{pmatrix} \mathbf{A} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \mathbf{R_{23}^{-1}} \\ &= \frac{1}{2E} \mathbf{R_{23}} \begin{bmatrix} \mathbf{R_{13}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \Delta m_{31}^2 \end{pmatrix} \mathbf{R_{13}^{-1}} + \begin{pmatrix} \mathbf{A} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \end{bmatrix} \mathbf{R_{23}^{-1}} \\ &= \frac{1}{2E} \mathbf{R_{23}} \begin{bmatrix} \begin{pmatrix} \bullet & 0 & \bullet \\ 0 & 0 & \bullet \\ \bullet & 0 & \bullet \end{pmatrix} + \begin{pmatrix} \mathbf{A} & 0 & 0 \\ 0 & 0 & 0 \\ \bullet & 0 & 0 \end{pmatrix} \end{bmatrix} \mathbf{R_{23}^{-1}} \\ &= \frac{1}{2E} \mathbf{R_{23}} \begin{bmatrix} \mathbf{R_{13}'} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 \\ \bullet & 0 & \Delta (m_{31}^2)' \end{pmatrix} (\mathbf{R_{13}'})^{-1} \end{bmatrix} \mathbf{R_{23}^{-1}} \end{split}$$

 $\Rightarrow$  re-insert  $R_{12} \Rightarrow U' \Rightarrow$  parameter mapping in 1-3 subspace

- Different mappings for neutrinos and antineutrinos
- 1-3 sub-space mapping like in 2 neutrino case

• Relevant quantitiv 
$$C_{\pm}^2 = \left(\frac{A}{\Delta m_{31}^2} - \cos 2\theta_{13}\right)^2 + \sin^2 2\theta_{13}$$

- MSW resonance condition for  $\theta_{13} \simeq 0$ :  $\Delta m_{31}^2 = A = 2VE = \pm \frac{2\sqrt{2}G_F Y \rho E}{m_n}$
- Effective parameters in matter:

$$\sin^{2} 2\theta'_{13} = \frac{\sin^{2} 2\theta_{13}}{C_{\pm}^{2}}$$
$$\Delta m_{31,m}^{2} = \Delta m^{2} C_{\pm}$$
$$\Delta m_{32,m}^{2} = \frac{\Delta m^{2} (C_{\pm} + 1) + A}{2}$$
$$\Delta m_{21,m}^{2} = \frac{\Delta m^{2} (C_{\pm} - 1) - A}{2}$$

• Corrections due to

$$-\Delta m_{12}^2 \neq 0$$

- non-constant matter profiles  $\Rightarrow$   $\,$  solve Schrödinger equation

## **Analytic Approximations**

$$\mathbf{P}(\mathbf{v}_{e} \rightarrow \mathbf{v}_{\mu}) = \\ \approx \quad \sin^{2} 2\theta_{13} \sin^{2} \theta_{23} \quad \frac{\sin^{2}((1-\hat{A})\Delta)}{(1-\hat{A})^{2}}$$

 $\Delta = \Delta m_{31}^2 L/4E$  $\alpha = \Delta m_{21}^2 / \Delta m_{31}^2 \sim 1/30$ A = matter potential

 $\sin \delta_{\rm CP} \,\alpha \sin 2\theta_{12} \cos \theta_{13} \sin 2\theta_{13} \sin 2\theta_{23} \sin(\Delta) \frac{\sin(\hat{A}\Delta) \sin((1-\hat{A})\Delta)}{\hat{A}(1-\hat{A})}$  $\pm$  $\cos \delta_{\rm CP} \alpha \sin 2\theta_{12} \cos \theta_{13} \sin 2\theta_{13} \sin 2\theta_{23} \cos(\Delta) \frac{\sin(\hat{A}\Delta) \sin((1-\hat{A})\Delta)}{\hat{A}(1-\hat{A})}$ 

+

+ 
$$\alpha^2 \sin^2 2\theta_{12} \cos^2 \theta_{23} \frac{\sin^2(\hat{A}\Delta)}{\hat{A}^2}$$

Cervera et al. Freund, Huber, ML Akhmedov, Johansson, ML, Ohlsson, Schwetz

## **Degeneracies, Correlations, ...**

#### Fixed L/E → probabilities invarinat under transformations:

- $\theta_{23} \rightarrow \pi/2 \theta_{23}$  Fogli, Lisi P( $v_e \rightarrow v_{\mu}$ ) not really invariant  $\rightarrow$  compensation by small parameter off-sets
- $\Delta m^2 \rightarrow -\Delta m^2$  compensated by offset in  $\delta$  Minakata, Nunokawa
- $P(v_e \rightarrow v_{\mu}) = const. \rightarrow \delta \theta_{13}$  manifolds Koike, Ota, Sato & Burguet-Castell et al.
- **>** 8-fold degeneracy Barger, Marfatia, Whisnant

- parameter extraction suffers from correlations & degeneracies
- how to break degeneracies & correlations?

## The magic Baseline

$$\begin{split} P(\nu_e \to \nu_\mu) &\approx \sin^2 2\theta_{13} \sin^2 \theta_{23} \ \frac{\sin^2((1-\hat{A})\Delta)}{(1-\hat{A})^2} \\ &\pm \ \sin \delta_{\rm CP} \ \alpha \sin 2\theta_{12} \cos \theta_{13} \sin 2\theta_{13} \sin 2\theta_{23} \sin(\Delta) \frac{\sin(\hat{A}\Delta) \sin((1-\hat{A})\Delta)}{\hat{A}(1-\hat{A})} \\ &+ \ \cos \delta_{\rm CP} \alpha \sin 2\theta_{12} \cos \theta_{13} \sin 2\theta_{13} \sin 2\theta_{23} \cos(\Delta) \frac{\sin(\hat{A}\Delta) \sin((1-\hat{A})\Delta)}{\hat{A}(1-\hat{A})} \\ &+ \ \alpha^2 \sin^2 2\theta_{12} \cos^2 \theta_{23} \frac{\sin^2(\hat{A}\Delta)}{\hat{A}^2} \end{split}$$

- All terms besides the first vanish for  $\sin(\hat{A}\Delta) = 0$
- Condition for uncorrelated sensitivity to  $\theta_{13}$   $\hat{A}\Delta = \pi$ 
  - $\Rightarrow$  inserting  $\hat{A}=A/\Delta m^2_{31}$ , A=2VE,  $\Delta=\Delta m^2_{31}L/4E$  one finds

$$L_{magic} = \frac{2\pi}{\sqrt{2}G_F n_e} = 7630 \text{ km} \cdot \frac{\rho}{4.3g/cm^3}$$
 Huber, Winter

• Note that this is not the MSW resonance condition
## **Status of Neutrino Oscillations**

