Beyond the Standard Model

Lecture #2: Supersymmetry

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why supersymmetry?

- **Supersymmetry (with the additional assumption of soft breaking at the TeV scale) has been the dominant BSM framework for the past 27 years.**

- **One reason is simply that SUSY is a *comprehensive* framework that is both *calculable* and *predictive*, making it possible to write 20,000 papers.**

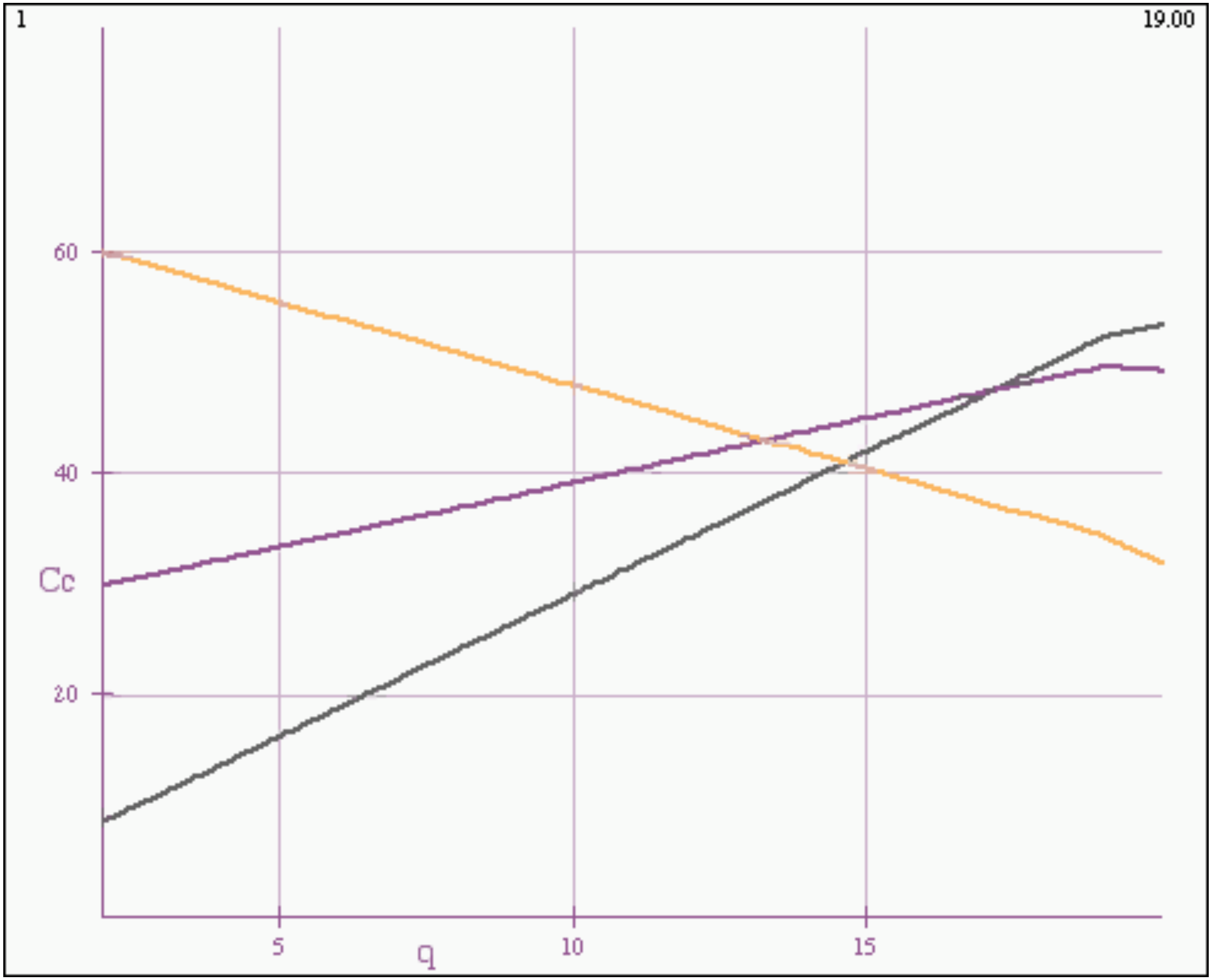
- **Another reason is that SUSY has many attractive theoretical properties, most especially that it suppresses quantum corrections, and is a space-time symmetry rather than an *ad hoc* postulation of extra degrees of freedom and "internal" symmetries.**

why supersymmetry?

- SUSY has also been criticized. Some of these complaints are valid, others are not.
- Invalid criticism of SUSY:
    - SUSY invents 137 new (so far never seen) particles and 137 new adjustable parameters. So it is not economical.
- Reply:
    - SUSY is the (more-or-less) unique extension of the known space-time symmetries. The new particles follow from this.
    - The “adjustable” parameters are only adjustable in the absence of a specified computable mechanism to break SUSY.

why supersymmetry?

- Invalid criticism of SUSY:
  - SUSY is pretty, but it is just as likely to be broken at the Planck scale as at the TeV scale.
- Reply:
  - Gauge coupling unification points to SUSY breaking at a lower scale, <~ 1000 TeV.
  - The measured dark matter density, if interpreted as a thermal relic WIMP, points to SUSY breaking at ~1 TeV.
  - SUSY is the only known robust method to stabilize the hierarchy between the electroweak scale and the Planck scale.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what are all the space-time symmetries of quantum physics?

- **We assume in quantum field theory that all interactions are local, Lorentz invariant, and translation invariant.**

- **We actually mean invariance under "proper orthochronous" Lorentz transformations, i.e. those that can be built up from infinitesimal ones.**

- **The parity operation** $P: \quad P\vec{x} \to -\vec{x}, \quad Pt \to t$ **is also a Lorentz transformation, but not all theories (e.g. the SM) are parity invariant.**

- **The same is true for the time reversal operator T.**

what are all the space-time symmetries of quantum physics?

- However every local field theory is invariant under the combined operation CPT.

- Thus every particle is either its own CPT conjugate (e.g. the photon), or it has a partner particle with the same mass, the same spin, but opposite charges.

$$CPT|m, s, s_z, q\rangle \to (-1)^{s-s_z}|m, s, -s_z, q\rangle$$

- We call these partners “antiparticles”.

what are all the space-time symmetries of quantum physics?

- **Are there any other spacetime symmetries consistent with local quantum field theory?**

- **One possibility is scale transformations.**

$$D: \quad D\vec{x} \to \lambda\vec{x}, \quad Dt \to \lambda t \qquad\qquad D|m,s,q\rangle \to |\lambda^{-1}m,s,q\rangle$$

- **Since D also rescales masses, D invariance means that either you only have massless particles, or every particle has a continuum of partners with different masses.**

- **The second possibility is not seen in nature.**

- **The first possibility, called scale invariance (conformal invariance), could at best be an approximate symmetry.**

what are all the space-time symmetries of quantum physics?

- **Are there any other space-time symmetries consistent with local quantum field theory?**

- **(With some mild caveats) the only other possibility is supersymmetry.**

$$Q_\alpha = (Q_1, Q_2)$$

$$\begin{aligned} Q_1^\dagger|m,s,s_z,q\rangle &\rightarrow \sqrt{2m}|m,s+\frac{1}{2},s_z+\frac{1}{2},q\rangle \\ Q_2^\dagger|m,s,s_z,q\rangle &\rightarrow \sqrt{2m}|m,s+\frac{1}{2},s_z-\frac{1}{2},q\rangle \end{aligned}$$

- **In a supersymmetric theory every particle has a “superpartner” with the same mass and charges, but spin differing by 1/2 hbar.**

antiparticles suppress quantum corrections

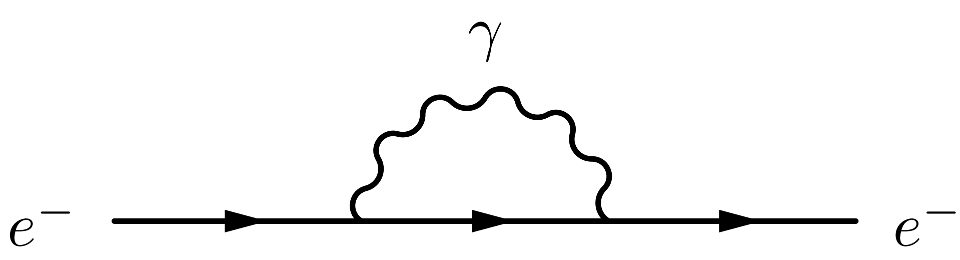
- **In QED the electron has a positive "self-energy" contribution to its mass, coming from the fact that it takes energy to squeeze an electric charge distribution down to a very small radius.**

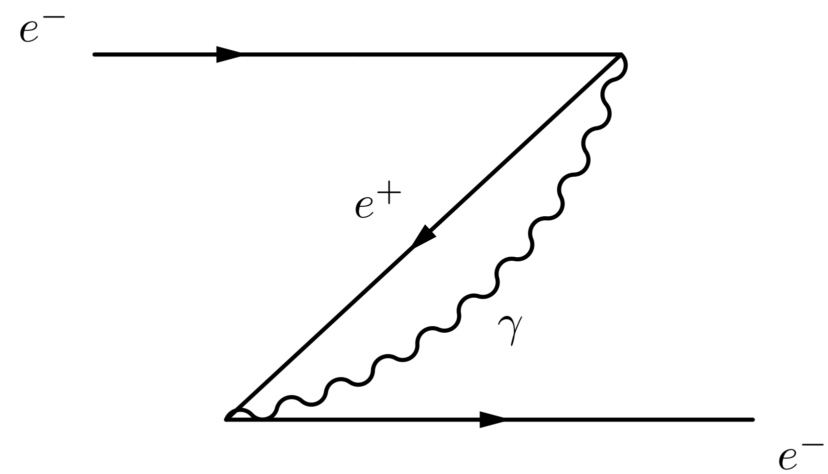
- **Classically (i.e. in the tree-level static limit of QED), this self-energy, from Coulomb's Law, is a linearly divergent function of the electric charge radius of the electron:**

$$\frac{\delta m_e}{m_e} = \frac{\alpha}{m_e r_e}$$

- **Since electrons look point-like up to energies >~ TeV, we know that**

$$m_e r_e <\sim 10^{-6}$$





$$-i\delta m_e + \ldots = (-ie)^2 \int \frac{d^4k}{(2\pi)^4} \frac{-i}{k^2} \left[\frac{\gamma^\mu i(\not{p} - \not{k} + m_e)\gamma_\mu}{(p-k)^2 - m_e^2} \right]$$

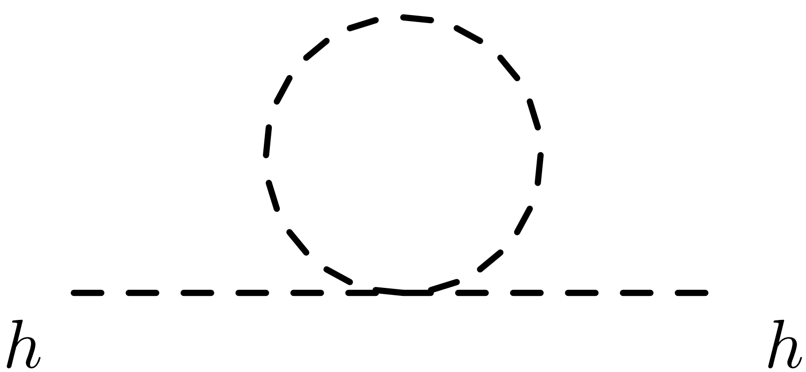
- **The second diagram is the correct interpretation of the contributions from** $E_p - E_k < 0$
- **Note this represents an interaction with virtual e+e- pairs in the vacuum.**
- **The net result is only logarithmically divergent:**

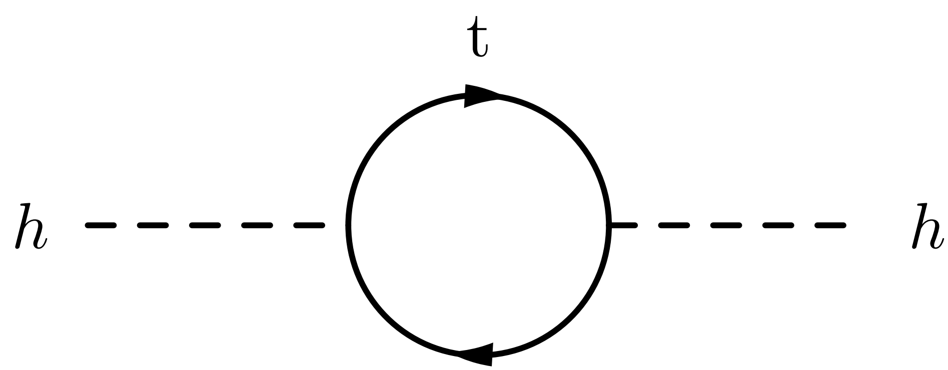
$$\frac{\delta m_e}{m_e} = -\frac{3}{2\pi}\alpha \log\left(m_e r_e\right) \qquad = 18\% \text{ for } r_e = 1/M_{\text{Planck}}$$

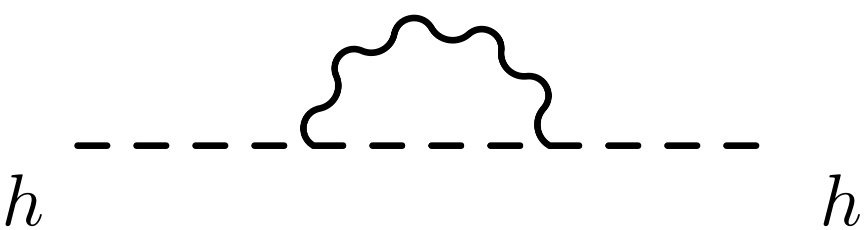
superpartners suppress quantum corrections

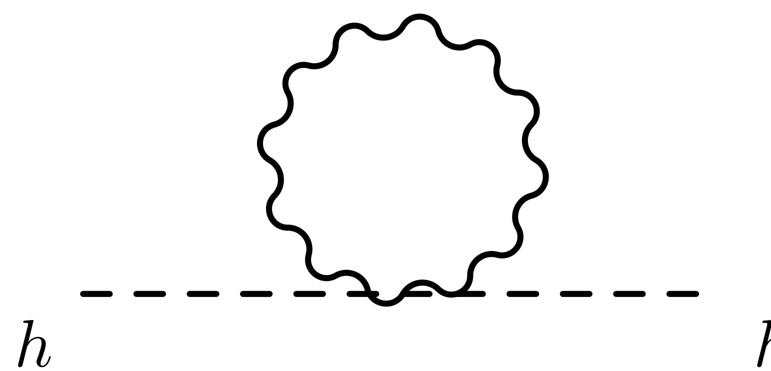
- **In the SM the Higgs particle has both positive and negative quantum corrections to its mass, coming from interactions with virtual quarks and leptons (mostly the top quark), W's, Z's, and the Higgs itself.**

$$\frac{\mathbf{3G_F}}{\mathbf{8\sqrt{2}}\pi^{\mathbf{2}}}\left(\mathbf{m_h^2} + \mathbf{2m_W^2} + \mathbf{m_Z^2} - \mathbf{4m_t^2}\right)\mathbf{\Lambda^2}$$

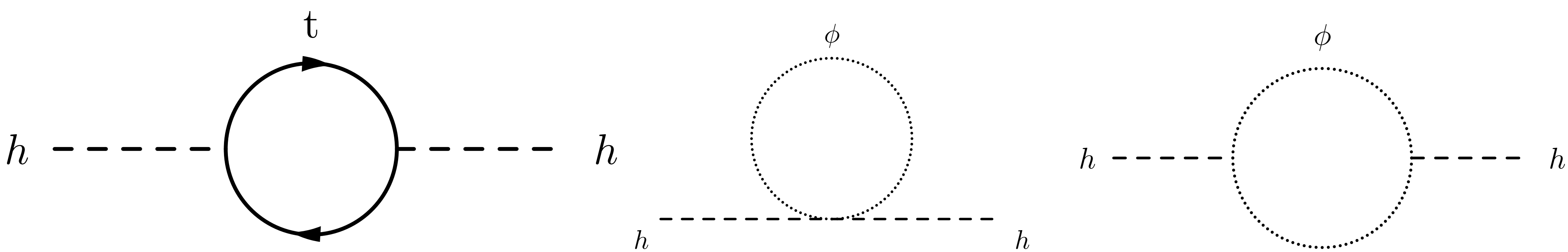




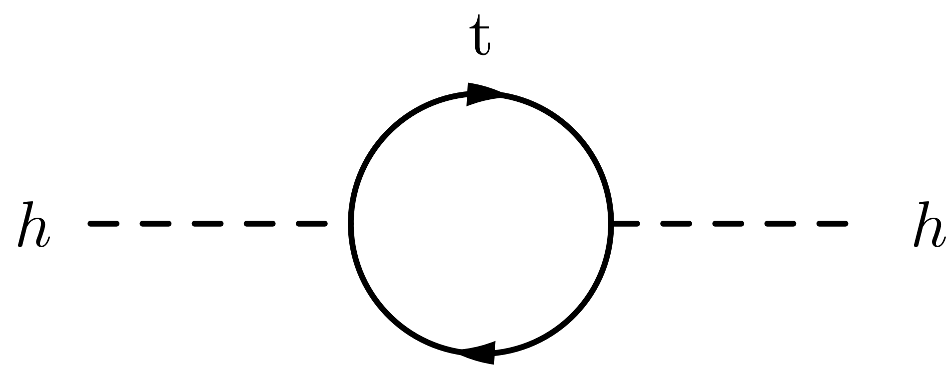




$$\mathcal{L} = (\text{kinetic terms}) - \frac{\lambda_t}{\sqrt{2}}(v+\mathbf{h})\bar{\mathbf{t}}_\mathbf{L}\mathbf{t}_\mathbf{R} - (\frac{\lambda}{\mathbf{2}}\mathbf{h}^2 + \mu\mathbf{h} + \mathbf{m}^2)(|\phi_\mathbf{L}|^2 + |\phi_\mathbf{R}|^2)$$



- **Add two complex scalar color triplets $\phi_\mathbf{L}, \phi_\mathbf{R}$ with quartic and cubic couplings to the Higgs, to try to cancel the divergent quantum corrections from the top quark loop**
- **For the moment, keep the coupling constants as free parameters**

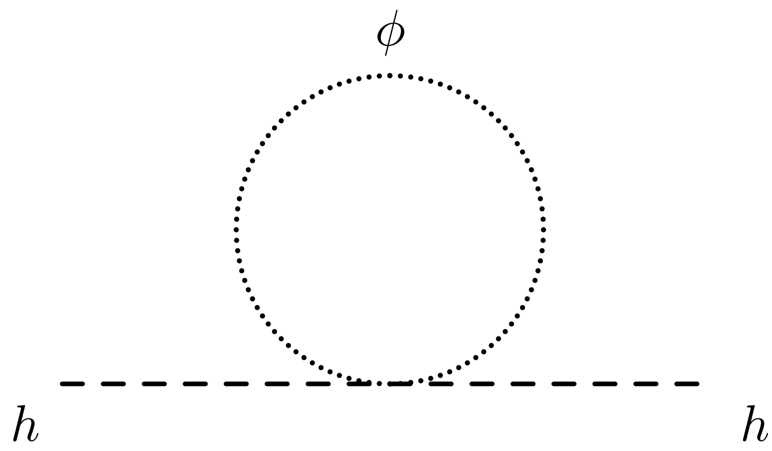


$$
\begin{aligned}
-i\delta m_h^2|_{\rm top} &= -N_c \left(\frac{-i\lambda_t}{\sqrt{2}}\right)^2 \int \frac{d^4k}{(2\pi)^4} \frac{i(\not{k}+m_t)i(\not{k}+m_t)}{(k^2-m_t^2)^2} \\
&= -6\lambda_t^2 \int \frac{d^4k}{(2\pi)^4} \frac{(k^2+m_t^2)}{(k^2-m_t^2)^2}
\end{aligned}
$$

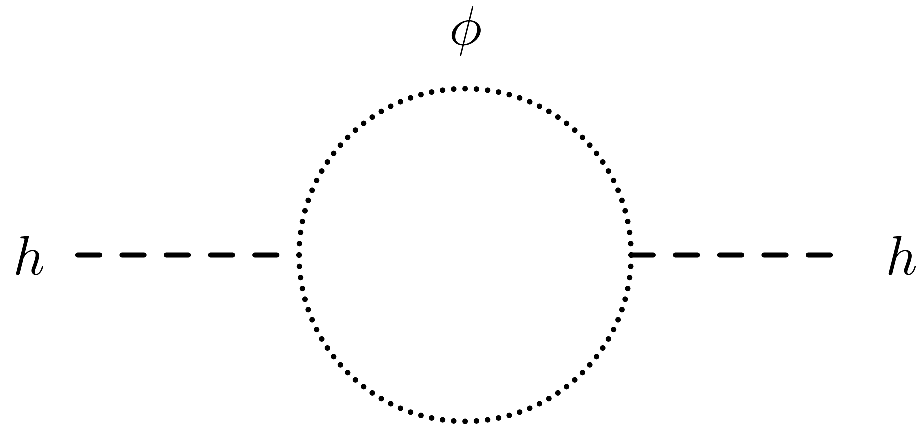
$$k_0 \to ik_4,\ k^2 \to -k_E^2$$

$$\int dk_E^4\ f(k_E^2) = 2\pi^2 \int k_E^3\, dk_E\ f(k_E^2) = \pi^2 \int k_E^2\, d(k_E^2)\ f(k_E^2)$$

$$
\begin{aligned}
\delta m_h^2|_{\rm top} &= -3\lambda_t^2 \int_0^{\Lambda^2} \frac{d(k_E^2)}{8\pi^2} \frac{k_E^2(k_E^2-m_t^2)}{(k_E^2+m_t^2)^2} = -\frac{3\lambda_t^2}{8\pi^2} \int_{m_t^2}^{\Lambda^2+m_t^2} dx\, \frac{(x-m_t^2)(x-2m_t^2)}{x^2} \\
&= -\frac{3\lambda_t^2}{8\pi^2} \left(\Lambda^2 - 3m_t^2 \log\left[\frac{\Lambda^2+m_t^2}{m_t^2}\right] + \dots\right)
\end{aligned}
$$

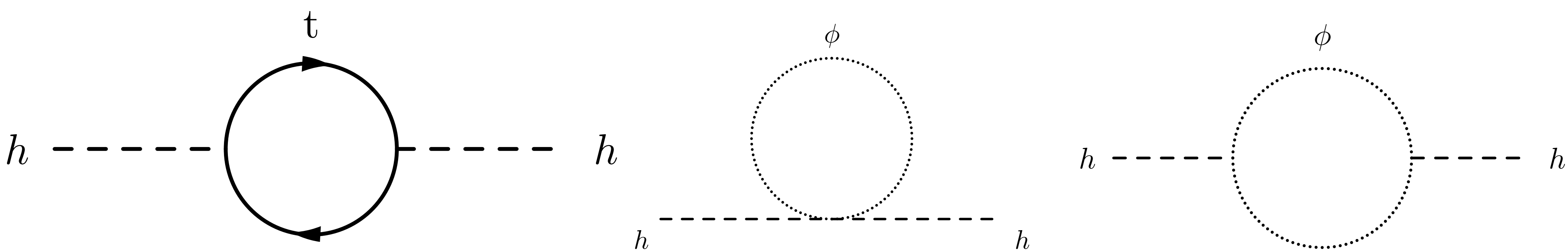


$$
\begin{aligned}
-i\delta m_h^2|_{\phi^2\mathrm{h}^2} &= -4i\frac{\lambda}{2}N_c\int\frac{d^4k}{(2\pi)^4}\frac{i}{k^2-m^2} \\
\delta m_h^2|_{\phi^2\mathrm{h}^2} &= 6i\lambda\int\frac{d^4k}{(2\pi)^4}\,\frac{1}{k^2-m^2}=\frac{3\lambda}{8\pi^2}\int d(k_E^2)\,\frac{k_E^2}{k_E^2+m^2} \\
&= \frac{3\lambda}{8\pi^2}\left(\Lambda^2-m^2\log\left[\frac{\Lambda^2+m^2}{m^2}\right]+\ldots\right)
\end{aligned}
$$



$$
\begin{aligned}
-i\delta m_h^2|_{\phi^2 \mathrm{h}} &= -2(i\mu)^2 N_c \int \frac{d^4k}{(2\pi)^4} \left(\frac{i}{k^2 - m^2}\right)^2 \\
\delta m_h^2|_{\phi^2 \mathrm{h}} &= 6i\mu^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m^2)^2} = -\frac{3\mu^2}{8\pi^2} \int d(k_E^2) \frac{k_E^2}{(k_E^2 + m^2)^2} \\
&= -\frac{3\mu^2}{8\pi^2} \left(\log\left[\frac{\Lambda^2 + m^2}{m^2}\right] + \ldots\right)
\end{aligned}
$$

$$\begin{aligned}\delta m_h^2 &= \delta m_h^2|_{\rm top} + \delta m_h^2|_{\phi^2 h^2} + \delta m_h^2|_{\phi^2 h} \\ &= -\frac{3\lambda_t^2}{8\pi^2}\left(\Lambda^2 - 3m_t^2 \log\left[\frac{\Lambda^2+m_t^2}{m_t^2}\right]\right) + \frac{3\lambda}{8\pi^2}\left(\Lambda^2 - m^2 \log\left[\frac{\Lambda^2+m^2}{m^2}\right]\right) - \frac{3\mu^2}{8\pi^2}\left(\log\left[\frac{\Lambda^2+m^2}{m^2}\right]\right)\end{aligned}$$



- **If we assume $\lambda = \lambda_t^2$, then the quadratically divergent corrections cancel.**
- **Notice this DOES NOT require that the masses are equal.**
- **However if in addition we have $m = m_t$ and $\mu^2 = 2\lambda_t^2 m_t^2$, then the log divergent pieces also cancel.**

superpartners suppress quantum corrections

- **In the SM the Higgs particle has both positive and negative quantum corrections to its mass, coming from interactions with virtual quarks and leptons (mostly the top quark), W's, Z's, and the Higgs itself.**

$$\frac{3G_F}{8\sqrt{2}\pi^2}\left(m_h^2 + 2m_W^2 + m_Z^2 - 4m_t^2\right)\Lambda^2$$

- **But if we introduce appropriate "superpartners" of the SM particles, and adjust the dimensionless couplings appropriately, we can cancel all quadratically divergent quantum corrections.**

- **Thus "softly-broken" SUSY solves the Higgs naturalness problem.**

- **Softly-broken means that the dimensionless couplings obey SUSY relations, but those with dimension of mass or mass-squared do not.**

superpartners:
squarks and sleptons

- Because SUSY is a space-time symmetry, it does not allow you to pick and choose which superpartners you want.

- Every SM particle has a superpartner.

- As we saw, for SM fermions the left-handed Weyl spinors, which couple to the W boson, have a complex scalar superpartner that couples to the W. The right-handed Weyl spinors have their own complex scalar superpartners. Thus

$$\mathbf{t_L} \leftrightarrow \tilde{\mathbf{t}}_{\mathbf{L}}, \quad \mathbf{t_R} \leftrightarrow \tilde{\mathbf{t}}_{\mathbf{R}}$$

- These spin 0 superpartners are called squarks and sleptons

gauginos

- The 8 spin 1 massless gluons of the SM have 8 massive spin 1/2 color octet superpartners called gluinos.

- These gluinos, like all gauginos, are Majorana fermions, because they are the superpartners of gauge bosons, which are their own antiparticles.

- The massive Majorana superpartners of the other SM gauge bosons are the 2 charged winos, the neutral wino, and the neutral bino.

higgsinos

- The SM has an SU(2) complex doublet Higgs field H with hypercharge 1.
- It’s superpartner would be a single SU(2) doublet Weyl fermion with hypercharge 1.
- These are the same quantum numbers as the conjugate of a left-handed SM lepton doublet.
- Such a theory has a gauge anomaly, i.e. the SM gauge symmetry would be broken by quantum effects.

SUSY requires (at least) two Higgs doublets

- The simplest solution to this problem is to have two complex doublet Higgs fields, Hu and Hd, with opposite hypercharges 1 and -1

- This avoids the anomaly because the new higgsino fermions are then a “vectorlike” pair.

- Hu has Yukawa couplings to up-type quarks only.

- Hd has Yukawa couplings to down-type quarks and leptons only.

SUSY requires (at least) two Higgs doublets

- **So there are TWO vevs that break electroweak symmetry, but still only 3 Goldstone bosons.**

- **The ratio of the up-type Higgs vev to the down-type Higgs vev is called** $\tan\beta$

$$\tan\beta = \frac{v_{\mathbf{u}}}{v_{\mathbf{d}}}$$

- **So instead of** $4 - 3 = 1$ **real Higgs boson, SUSY requires at least** $8 - 3 = 5$ **Higgs bosons:**

$$\mathbf{h},\ \mathbf{H},\ \mathbf{A},\ \mathbf{H}^{\pm}$$

higgsinos

- **Having expanded the Higgs sector, we see that we get 4 higgsinos, corresponding to the total of 4 complex scalars from two Higgs doublets.**

- **Two higgsinos are neutral and two are charged.**

- **Because these are the superpartners of a vectorlike pair of charged boson doublets, we can write all of these higgsinos in Majorana combinations.**

- **Now we should worry that the 2 neutral higgsinos might mix with the neutral wino and bino, and the 2 charged higgsinos might mix with the 2 charged winos.**

charginos

- Indeed, the gauge covariant kinetic term of a Higgs doublet has couplings of a W to a charged Higgs and a neutral Higgs. $\mathbf{D}^{\mu}\mathbf{H}^{\dagger}\mathbf{D}_{\mu}\mathbf{H}$

- So the supersymmetrized version has a coupling of a charged wino to a charged higgsino and a neutral Higgs vev.

- This bilinear fermion coupling, after EW symmetry breaking, mixes the winos with the charged higgsinos.

- For experiments we only care about the resulting mass eigenstates, called the lighter charginos $\tilde{\chi}^{\pm}_{\mathbf{1}}$ and the heavier charginos $\tilde{\chi}^{\pm}_{\mathbf{2}}$

neutralinos

- Similarly, after EW symmetry breaking there are bilinear couplings that mix the 2 neutral higgsinos with the neutral wino and bino.

- The resulting 4 mass eigenstates are called neutralinos. Like the charginos they are numbered in order of lightest to heaviest:

$$\tilde{\chi}_1^0,\ \tilde{\chi}_2^0,\ \tilde{\chi}_3^0,\ \tilde{\chi}_4^0$$

superpartner standard notation

spin $\frac{1}{2}$ Majorana fermion gauginos+higgsinos:

- color octet gluino: $\tilde{g}$
- mass eigenstate mixtures of wino and charged higgsino: $\tilde{\chi}_1^\pm$, $\tilde{\chi}_2^\pm$
- mass eigenstate mixtures of photino, bino, and two neutral higgsinos: $\tilde{\chi}_1^0$, $\tilde{\chi}_2^0$, $\tilde{\chi}_3^0$, $\tilde{\chi}_4^0$

spin 0 complex scalar squarks:

- squarks that couple to the W boson: $\tilde{u}_L$, $\tilde{d}_L$, $\tilde{c}_L$, $\tilde{s}_L$
- squarks that do not couple to the W boson: $\tilde{u}_R$, $\tilde{d}_R$, $\tilde{c}_R$, $\tilde{s}_R$
- mass eigenstate mixtures of $\tilde{t}_L$ and $\tilde{t}_R$: $\tilde{t}_1$, $\tilde{t}_2$
- mass eigenstate mixtures of $\tilde{b}_L$ and $\tilde{b}_R$: $\tilde{b}_1$, $\tilde{b}_2$

spin 0 complex scalar sleptons:

- sleptons that couple to the W boson: $\tilde{e}_L$, $\tilde{\mu}_L$, $\tilde{\nu}_e$, $\tilde{\nu}_\mu$, $\tilde{\nu}_\tau$
- sleptons that do not couple to the W boson: $\tilde{e}_R$, $\tilde{\mu}_R$
- mass eigenstate mixtures of $\tilde{\tau}_L$ and $\tilde{\tau}_R$: $\tilde{\tau}_1$, $\tilde{\tau}_2$

The MSSM

- **The supersymmetrized version of the SM, with *general* soft-breaking and the *minimally* extended Higgs sector just described is called the Minimal Supersymmetric Standard Model (MSSM).**

- **The MSSM is "minimal" only in its choice of Higgs sector. It is actually *maximal* in its choice of all possible soft-breaking interactions (105 new parameters!)**

- **This is an effective action approach to SUSY at the TeV scale, NOT a top-down theory of SUSY breaking starting from e.g. an elegant unified theory at the Planck scale.**

SUSY models

- There are an infinite number of models that implement supersymmetry softly broken at the TeV scale.

- A “complete” SUSY model will include an explicit mechanism for SUSY-breaking, and an explicit connection between the breaking of SUSY and of electroweak symmetry.

- Compared to the MSSM, these models may have new particles, new gauge interactions, a further expanded Higgs sector, etc.

- A more “complete” SUSY model will tend to have fewer adjustable parameters than the MSSM, and will thus be more predictive.

radiative electroweak breaking

- I claimed that SUSY models solve the Higgs naturalness problem, by suppressing the quadratically divergent quantum corrections to the Higgs mass.

- This suppression is enough to stabilize the hierarchy between the Planck scale (GUT scale, string scale, etc) and the scale of electroweak symmetry breaking.

- The remaining logarithmic effects cause the two Higgs mass-squared parameters $m^2_{H_u}$, $m^2_{H_d}$ to run with energy scale.

radiative electroweak breaking

- **The main effect on their running is from top loops.**

- **As we saw, the effect is proportional to the square of the top quark mass, and wants to drive the Higgs mass-squared parameters *negative*.**

- **Thus in softly-broken SUSY, we can start with *positive* Higgs mass-squared parameters, and we get electroweak symmetry breaking for free, triggered by the fact that the top quark is heavy.**

- **This is a remarkable feature of the SUSY framework.**

are SUSY models natural?

- **Softly-broken SUSY models have lots of scalars besides the Higgs, and lots of explicit soft-breaking mass parameters.**

- **All of these have the nice feature that their running is only logarithmic.**

- **The MSSM and many other SUSY models also contain an additional mass parameter μ, coming from a *supersymmetric* coupling of the two Higgs/higgsino doublets.**

- **Such models are NOT natural, if there is no mechanism to ensure that μ has a value related to the TeV scale values of the soft-breaking terms (and thus of the electroweak scale).**

R-parity

- In the SM baryon number (B) and lepton number (L) are automatically conserved (ignoring neutrinos and nonperturbative effects).

- Not so in SUSY, where one can write a bunch of dimension 4 operators that violate B and/or L.

- An example if a SM lepton doublet coupled to a SM quark doublet and a down-type squark:

$$\lambda^{\mathbf{ijk}} \mathbf{L_i} \mathbf{Q_j} \tilde{\mathbf{d}}^*_{\mathbf{k}}$$

- The squark has baryon number but not lepton number, so this interaction violates lepton number by 1 unit.

R-parity

- **Some combinations of these couplings would allow unsuppressed proton decay, a complete disaster.**
- **However all of these B and L violating couplings can be forbidden by a discrete symmetry, called R-parity.**
- **The SM particles are all even under R-parity, but all the superpartners are odd under R-parity.**
- **Obviously this forbids couplings like** $\lambda^{ijk} L_i Q_j \tilde{d}_k^*$

R-parity

- Conservation of R-parity means that superpartners can only be produced in pairs at colliders, since the initial state is R-parity even.

- It also implies that the lightest superpartner (LSP) is absolutely stable.

- Often the LSP turns out to be the lightest neutralino, which is a good WIMP dark matter candidate.

- SUSY models with stable *charged* LSPs are strongly disfavored by experimental/observational data.

superpartner pair production at the LHC

- In proton-proton collisions at LHC we expect squark and /or gluino production to dominate superpartner production, provided that their masses are less than ~ 2 TeV.
- The cross sections vary from ~10 fb to as as large as ~10 pb, for masses in the range ~ 600 - 2000 GeV.
- The strong dependence on the mass is partly because for heavy superpartners we are fighting against the steeply falling parton distribution functions (pdfs).

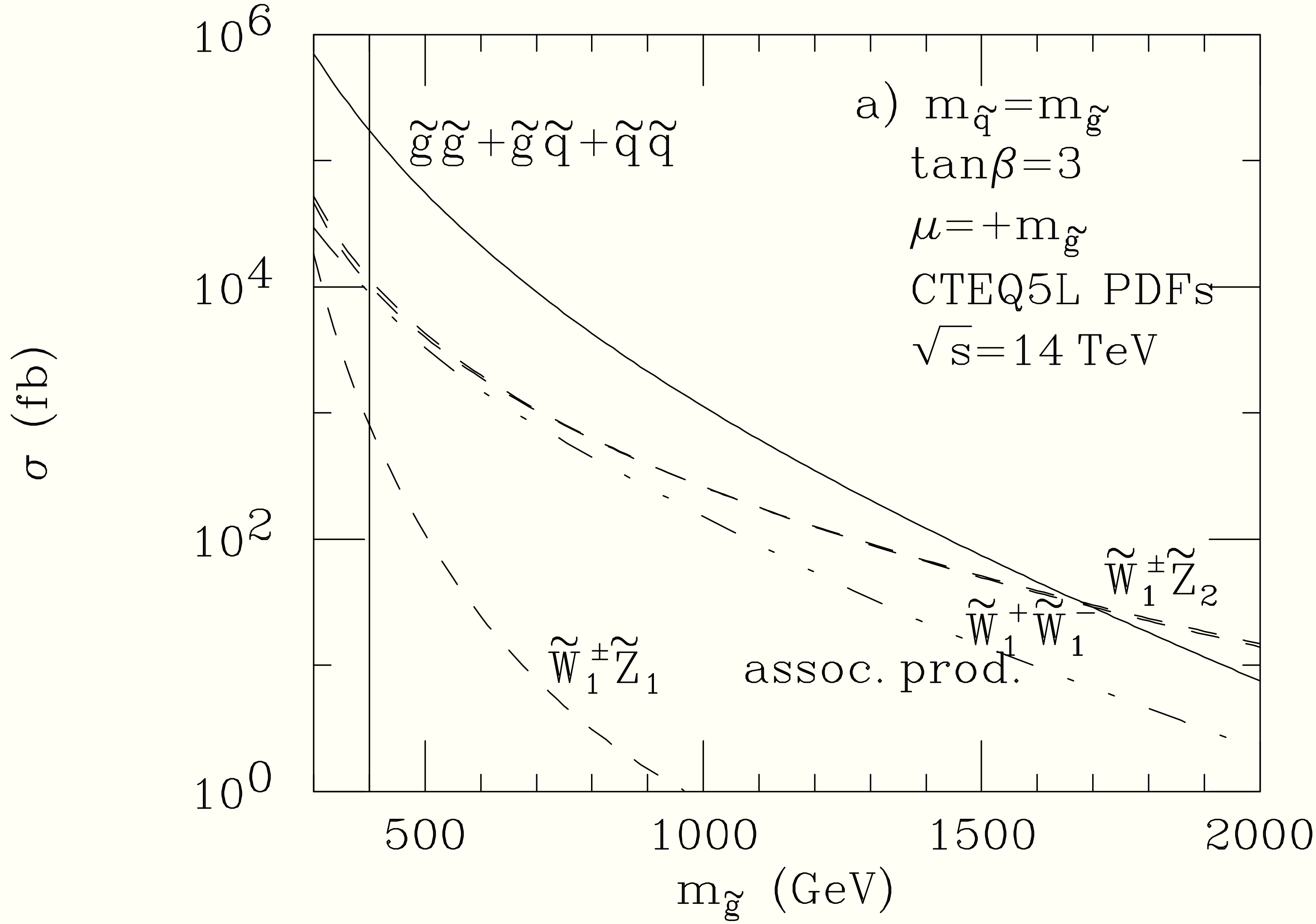

a) $m_{\tilde{q}}=m_{\tilde{g}}$
$\tan\beta=3$
$\mu=+m_{\tilde{g}}$
CTEQ5L PDFs
$\sqrt{s}=14$ TeV
$\tilde{g}\tilde{g}+\tilde{g}\tilde{q}+\tilde{q}\tilde{q}$
$\widetilde{W}_1^{\pm}\widetilde{Z}_2$
$\widetilde{W}_1^{+}\widetilde{W}_1^{-}$
$\widetilde{W}_1^{\pm}\widetilde{Z}_1$
assoc. prod.
σ (fb)
$m_{\tilde{g}}$ (GeV)
10^6
10^4
10^2
10^0
500
1000
1500
2000

Table 13.2. Cross sections for the test points in pb at NLO (LO) from PROSPINO1.

Point	$M(\tilde{q})$	$M(\tilde{g})$	$\tilde{g}\tilde{g}$	$\tilde{g}\tilde{q}$	$\tilde{q}\bar{\tilde{q}}$	$\tilde{q}\tilde{q}$	Total
LM1	558.61	611.32	10.55	28.56	8.851	6.901	54.86
			(6.489)	(24.18)	(6.369)	(6.238)	(43.28)
LM2	778.86	833.87	1.443	4.950	1.405	1.608	9.41
			(0.829)	(3.980)	(1.013)	(1.447)	(7.27)
LM3	625.65	602.15	12.12	23.99	4.811	4.554	45.47
			(7.098)	(19.42)	(3.583)	(4.098)	(34.20)
LM4	660.54	695.05	4.756	13.26	3.631	3.459	25.11
			(2.839)	(10.91)	(2.598)	(3.082)	(19.43)
LM5	809.66	858.37	1.185	4.089	1.123	1.352	7.75
			(0.675)	(3.264)	(0.809)	(1.213)	(5.96)
LM6	859.93	939.79	0.629	2.560	0.768	0.986	4.94
			(0.352)	(2.031)	(0.559)	(0.896)	(3.84)
LM7	3004.3	677.65	6.749	0.042	0.000	0.000	6.79
			(3.796)	(0.028)	(0.000)	(0.000)	(3.82)
LM8	820.46	745.14	3.241	6.530	1.030	1.385	12.19
			(1.780)	(5.021)	(0.778)	(1.230)	(8.81)
LM9	1480.6	506.92	36.97	2.729	0.018	0.074	39.79
			(21.44)	(1.762)	(0.015)	(0.063)	(23.28)
LM10	3132.8	1294.8	0.071	0.005	0.000	0.000	0.076
			(0.037)	(0.004)	(0.000)	(0.000)	(0.041)
HM1	1721.4	1885.9	0.002	0.018	0.005	0.020	0.045
			(0.001)	(0.016)	(0.005)	(0.021)	(0.043)
HM2	1655.8	1785.4	0.003	0.027	0.008	0.027	0.065
			(0.002)	(0.024)	(0.007)	(0.028)	(0.061)
HM3	1762.1	1804.4	0.003	0.021	0.005	0.018	0.047
			(0.002)	(0.018)	(0.004)	(0.019)	(0.043)
HM4	1815.8	1433.9	0.026	0.056	0.003	0.017	0.102
			(0.014)	(0.043)	(0.003)	(0.017)	(0.077)

squark production at the LHC

- Let's look in more detail at the case of squark pair production at LHC.
- I will make three simplifying assumptions:
  - Tree level diagrams only.
  - The gluino is very heavy (say, 4 TeV), so I don't have to compute squark production diagrams with t-channel exchanges of virtual gluinos.
  - I will assume that the squark flavor differs from the flavor of initial state quarks, which eliminates some additional production diagrams.

squark production at the LHC

- You already know how to compute LHC production of a pair of heavy quarks (don't you?), so I will compare my answers from squark pair production to this.

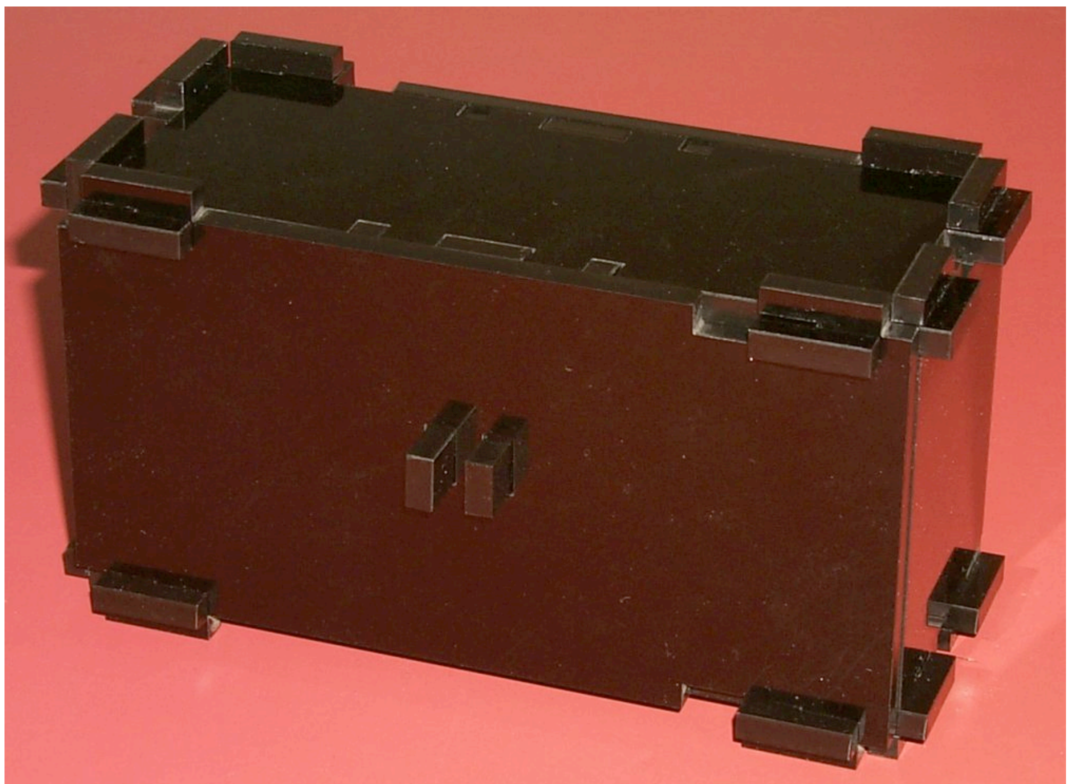
- You can look up heavy quark production in Ellis, Sterling and Webber (ESW).

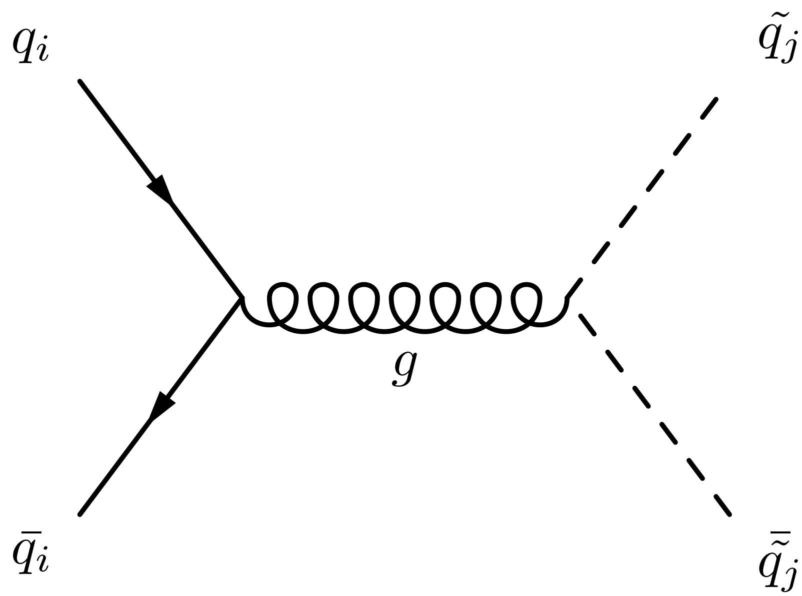
- Given my simplifying assumption about having distinct quark/squark flavors, there are only two partonic initial states that we have to worry about:

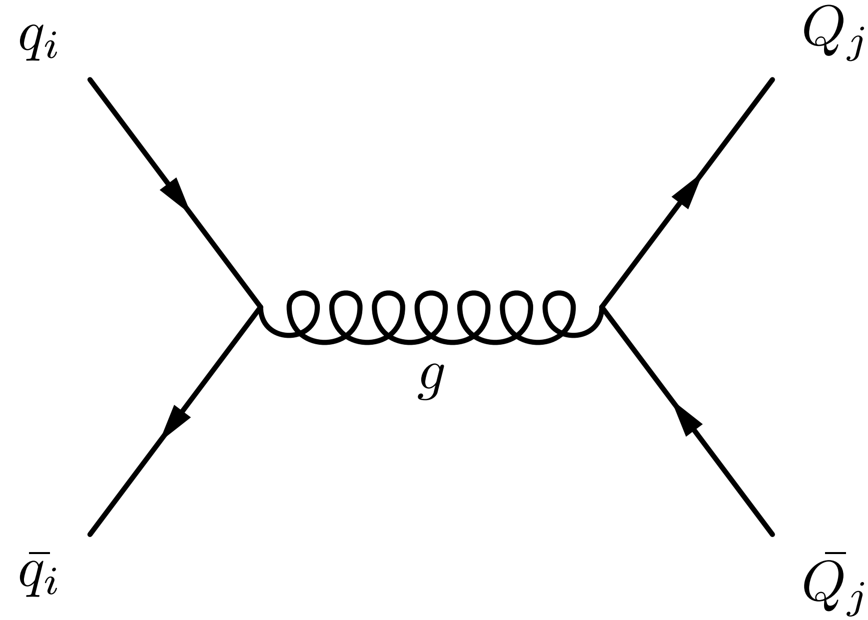
$$\begin{aligned} q_i \bar{q}_i &\rightarrow \tilde{q}_j \bar{\tilde{q}}_j \\ gg &\rightarrow \tilde{q}_j \bar{\tilde{q}}_j \end{aligned}$$

Beware of the Black Box!

- I could do all of this much more quickly with Pythia (or MadGraph, CompHEP, Prospino, ...)
- But physicists should not be 100% dependent on black boxes...







- **This is the simplest case since there is only one diagram.**
- **The initial state quark carries some fraction x_1 of its parent proton's energy, and the antiquark carries some other fraction x_2 of its parent proton's energy.**
- **We measure things in the lab frame, but we compute in the partonic subprocess center of mass frame.**

4-momenta in the subprocess CM frame:

$$
\begin{aligned}
p_q &= \frac{1}{2}\sqrt{\hat{s}}\,(1,0,0,1) \\
p_{\bar{q}} &= \frac{1}{2}\sqrt{\hat{s}}\,(1,0,0,-1) \\
p_{\tilde{q}} &= \frac{1}{2}\sqrt{\hat{s}}\,(1,\beta\sin\theta,0,\beta\cos\theta) \\
p_{\bar{\tilde{q}}} &= \frac{1}{2}\sqrt{\hat{s}}\,(1,-\beta\sin\theta,0,-\beta\cos\theta)
\end{aligned}
$$

subprocess CM energy squared:

$\hat{s} = x_1 x_2\, s$

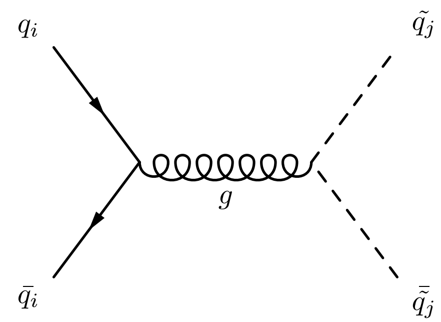
subprocess CM boost variables:

$$\beta = \sqrt{1-\frac{4m^2}{\hat{s}}} = \sqrt{1-\frac{1}{\gamma^2}}$$

Mandelstam invariant kinematic variables:

$$
\begin{aligned}
\hat{s} &= (p_q + p_{\bar{q}})^2 = (p_{\tilde{q}} + p_{\bar{\tilde{q}}})^2 = 4m^2\gamma^2 \\
\hat{t} &= (p_q - p_{\tilde{q}})^2 = (p_{\bar{q}} - p_{\bar{\tilde{q}}})^2 = m^2 - 2p_q\cdot p_{\tilde{q}} = m^2 - \frac{\hat{s}}{2}(1-\beta\cos\theta) \\
\hat{u} &= (p_q - p_{\bar{\tilde{q}}})^2 = (p_{\bar{q}} - p_{\tilde{q}})^2 = m^2 - 2p_q\cdot p_{\bar{\tilde{q}}} = m^2 - \frac{\hat{s}}{2}(1+\beta\cos\theta)
\end{aligned}
$$

$\hat{s} + \hat{t} + \hat{u} = 2m^2$



compute the squared matrix element, averaged over initial spins+colors, summed over final spins+colors

$$i\mathcal{M} = \bar{v}^{\bar{s}}(p_{\bar{q}})[-ig_s(t^a)_{ii'}\gamma_\mu]u^s(p_q)\left(\frac{-i}{\hat{s}}\right)[-ig_s(t^a)_{jj'}(p^\mu_{\tilde{q}} - p^\mu_{\bar{\tilde{q}}})]$$

$$\begin{aligned}\overline{\Sigma}|\mathcal{M}|^2 &= \left(\frac{1}{4}\right)\left(\frac{2}{9}\right)\frac{g_s^4}{\hat{s}^2}\,\mathrm{tr}\left[\not{p}_{\bar{q}}(\not{p}_{\tilde{q}} - \not{p}_{\bar{\tilde{q}}})\;\not{p}_q(\not{p}_{\tilde{q}} - \not{p}_{\bar{\tilde{q}}})\right]\\ &= \frac{1}{9}\,g_s^4\,\beta^2(1-\cos^2\theta)\\ &= \frac{2}{9}\,g_s^4\left[1-\tau_1^2-\tau_2^2-\frac{2m^2}{\hat{s}}\right]\end{aligned}$$

HOMEWORK

where we use the ESW invariant variables:

$$\begin{aligned}\tau_1 &= \frac{m^2-\hat{t}}{\hat{s}} = \frac{1}{2}(1-\beta\cos\theta)\\ \tau_2 &= \frac{m^2-\hat{u}}{\hat{s}} = \frac{1}{2}(1+\beta\cos\theta)\end{aligned}$$

$$\tau_1+\tau_2 = 1$$

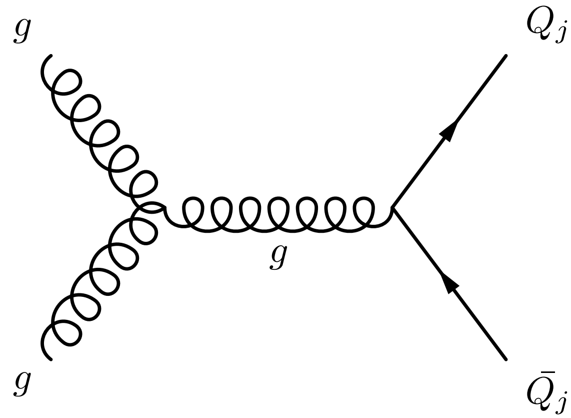
$$\frac{d^3\hat{\sigma}}{dx_1 dx_2 d\cos\theta} \quad = \quad 2\pi\frac{\beta}{64\pi^2\hat{s}} f_q(x_1) f_{\bar{q}}(x_2) \overline{\Sigma}|\mathcal{M}|^2 = \frac{\pi\beta\alpha_s^2}{9\hat{s}} f_q(x_1) f_{\bar{q}}(x_2) \left[1 - \tau_1 - \tau_2 - \frac{2m^2}{\hat{s}}\right]$$

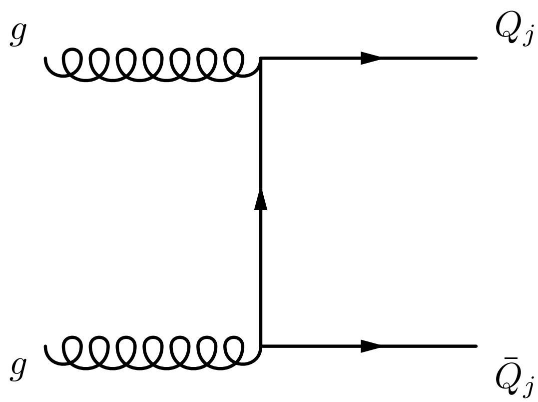
change variables, using the Jacobian:

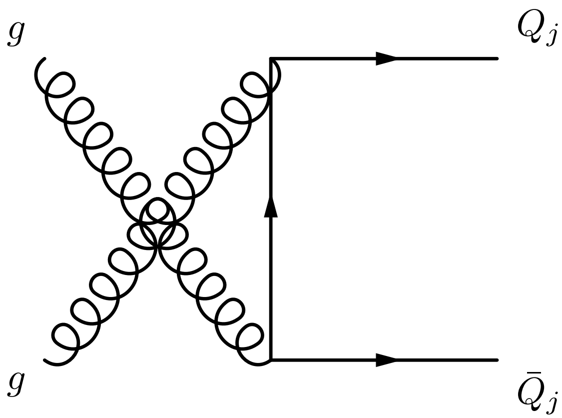
$$\frac{d(\cos\theta)}{d\hat{t}} = \frac{2}{\beta\hat{s}}$$

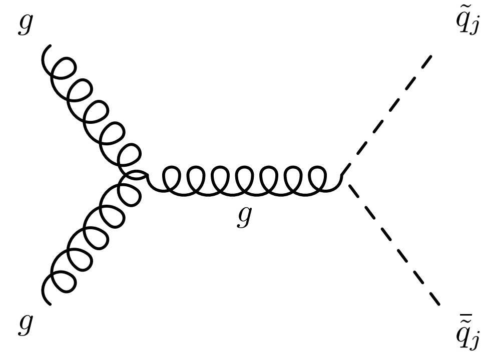
$$\frac{d^3\hat{\sigma}}{dx_1 dx_2 d\hat{t}} \quad = \quad \frac{2\pi\alpha_s^2}{9\hat{s}^2} f_q(x_1) f_{\bar{q}}(x_2) \left[1 - \tau_1 - \tau_2 - \frac{2m^2}{\hat{s}}\right]$$

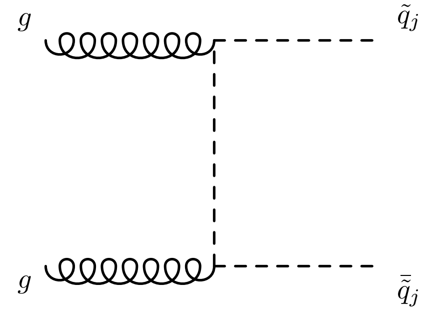
That wasn't so hard. Now let's do $\mathbf{gg -> \tilde{q}_j \bar{\tilde{q}}_j}$

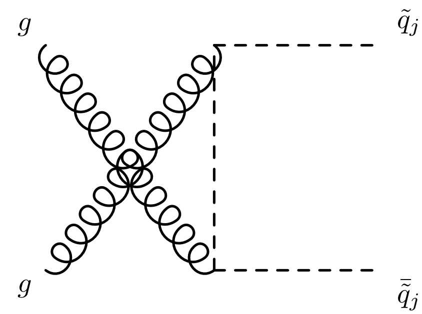




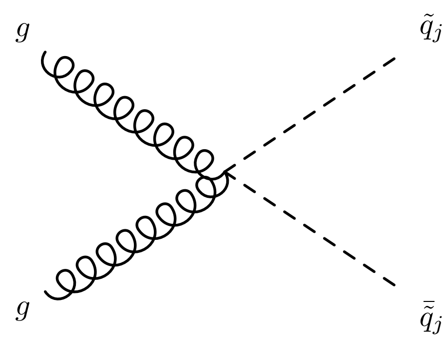








why do the squarks have an extra diagram?



Final answers:

$$\mathbf{q_i \bar{q}_i -> \tilde{q}_j \bar{\tilde{q}}_j}$$

$$\frac{d^3\hat{\sigma}}{dx_1 dx_2 d\hat{t}} = \frac{2\pi\alpha_s^2}{9\hat{s}^2} f_q(x_1) f_{\bar{q}}(x_2) \left[1 - \tau_1 - \tau_2 - \frac{2m^2}{\hat{s}}\right]$$

$$\mathbf{gg -> \tilde{q}_j \bar{\tilde{q}}_j}$$

$$\frac{d^3\hat{\sigma}}{dx_1 dx_2 d\hat{t}} = \frac{\pi\alpha_s^2}{2\hat{s}^2} f_q(x_1) f_{\bar{q}}(x_2) \left(\frac{1}{6\tau_1\tau_2} - \frac{3}{8}\right) \left[1 - \tau_1^2 - \tau_2^2 - \frac{4m^2}{\hat{s}} + \frac{4m^4}{\tau_1\tau_2\hat{s}^2}\right]$$

Compare to heavy quark production formulae from ESW:

$$\frac{d^3\hat{\sigma}}{dx_1 dx_2 d\hat{t}} = \frac{4\pi\alpha_s^2}{9\hat{s}^2} f_q(x_1) f_{\bar{q}}(x_2) \left[\tau_1 + \tau_2 + \frac{2m^2}{\hat{s}}\right]$$

$$\frac{d^3\hat{\sigma}}{dx_1 dx_2 d\hat{t}} = \frac{\pi\alpha_s^2}{\hat{s}^2} f_q(x_1) f_{\bar{q}}(x_2) \left(\frac{1}{6\tau_1\tau_2} - \frac{3}{8}\right) \left[\tau_1^2 + \tau_2^2 + \frac{4m^2}{\hat{s}} - \frac{4m^4}{\tau_1\tau_2\hat{s}^2}\right]$$

- **To make these results useful, we need to do two things:**
  - **Change variables to $\mathbf{p_T}$, $\eta_{\tilde{\mathbf{q}}}$, $\eta_{\bar{\tilde{\mathbf{q}}}}$, lab frame quantities that we would actually measure (or, in principle, reconstruct from the squark decay products).**
  - **Integrate over the parton distribution functions.**

$$\begin{aligned}\sinh \eta_{\tilde{q}} &= \frac{\sinh y_{\tilde{q}}}{p_T}\sqrt{p_T^2 + m^2} \\ \sinh \eta_{\bar{\tilde{q}}} &= \frac{\sinh y_{\bar{\tilde{q}}}}{p_T}\sqrt{p_T^2 + m^2}\end{aligned}$$

$$Y = \frac{1}{2}(y_{\tilde{q}} + y_{\bar{\tilde{q}}}), \quad \cosh^2[\frac{1}{2}(y_{\tilde{q}} - y_{\bar{\tilde{q}}})] = \frac{4}{\hat{s}}(p_T^2 + m^2)$$

$$x_1 = \sqrt{\frac{\hat{s}}{s}}\mathrm{e}^{Y}, \quad x_2 = \sqrt{\frac{\hat{s}}{s}}\mathrm{e}^{-Y}$$

- Suppose we want to know the pT distributions of the squarks.

- We can get these from our fully differential cross sections by changing variables to $\mathbf{p_T},\ \mathbf{x_1},\mathbf{x_2}$ and then integrating over $\mathbf{x_1}$ and $\mathbf{x_2}$, keeping the pT fixed.

- Using *Mathematica* you can do this explicitly using available packages for the CTEQ pdfs.

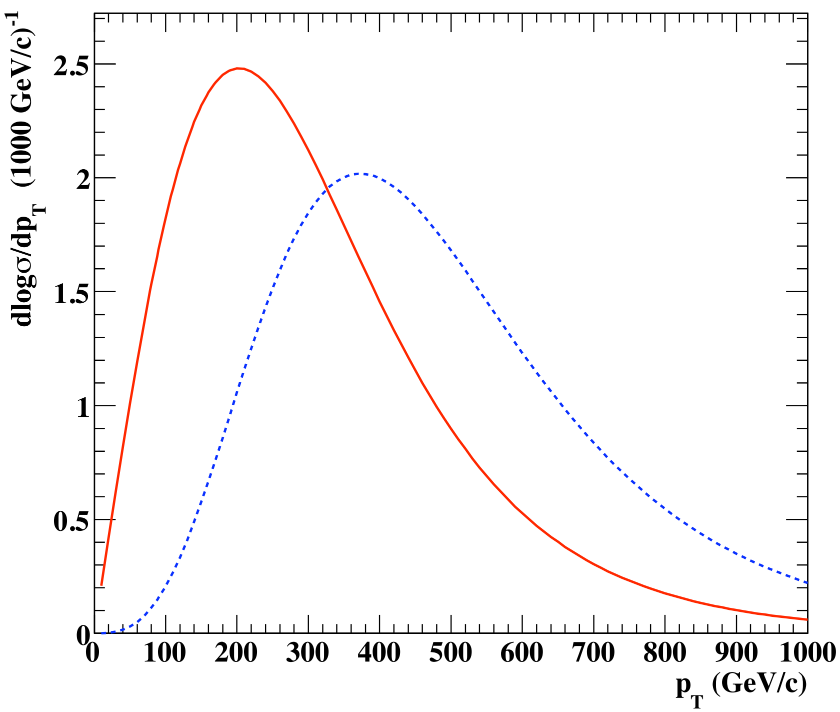
- You have to be careful about the integration limits:

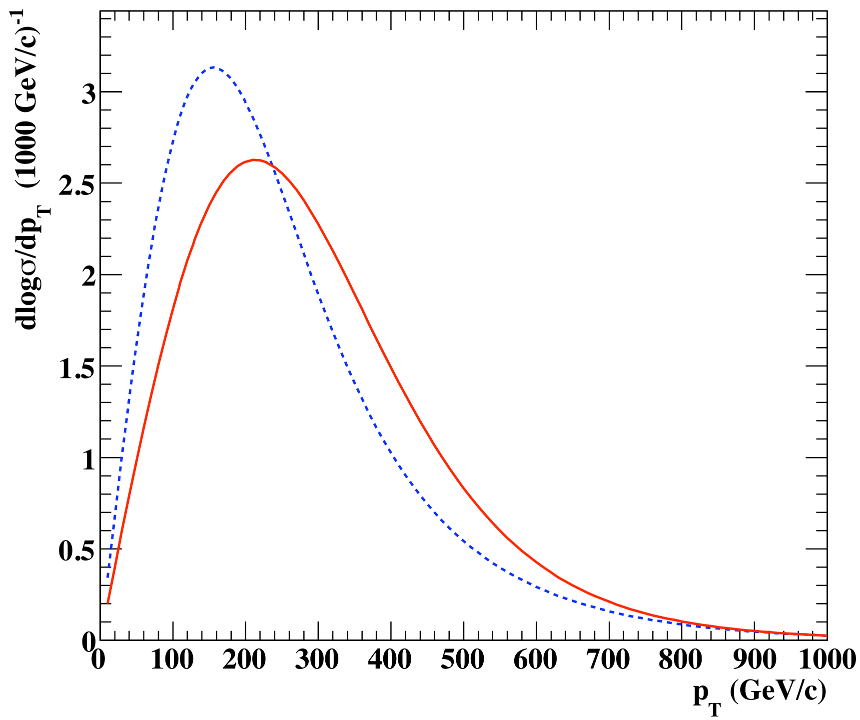
$$\int_0^{p_T^{\max}} dp_T \int_{X_{\min}}^1 dX \int_{\frac{1}{2}\log X}^{-\frac{1}{2}\log X} dY$$

$$X = x_1 x_2 \qquad Y = \log\frac{x_1}{\sqrt{X}} = -\log\frac{x_2}{\sqrt{X}}$$

$$X_{\min} = \frac{4}{s}(p_T^2 + m^2) \qquad\qquad p_T^{\max} = \sqrt{\frac{1}{4}s - m^2}$$

Compare pT spectra of squarks versus heavy quarks





$q\bar{q}$ initial state

gg initial state

m=500 GeV

red line = heavy quark
blue line = squark

cross sections and pT spectra depend on the spin of the particle that you produced!

Do we agree with the Monte Carlo?

CTEQ5L comparision:

```
ISUB (277) :
 PYTHIA :    213 fb            Us :   212.8  fb          Discrepancy :   0.1 %

ISUB (279) :
 PYTHIA :  1216 fb            Us :  1190    fb          Discrepancy :   2 %
```

CTEQ5M1 comparison:

```
ISUB (277) :
 PYTHIA :    199 fb            Us :   194   fb          Discrepancy :   3 %

ISUB (279) :
 PYTHIA :  1330 fb            Us :  1325   fb          Discrepancy :   0.4 %
```

Warning: to get agreement this good you must match the pdfs, the choice of scale, the definition of α_s running, etc.

Summary of second lecture - I

- SUSY is the dominant BSM framework, partly because it is comprehensive, calculable, and predictive.
- SUSY is a “unique” extension of the known space-time symmetries.
- SUSY predicts superpartners for every SM particle.
- SUSY suppresses quantum effects.
- SUSY solves the Higgs naturalness problem, even if “softly” broken.
- SUSY requires (at least) adding an extra Higgs doublet.
- The MSSM is an effective theory of softly-broken SUSY with the minimal particle content and the maximal soft-breaking interactions.

Summary of second lecture - II

- **Softly-broken SUSY breaks electroweak symmetry automatically via quantum effects involving the heavy top quark.**

- **Most SUSY models have a supersymmetric “mu-term” that reintroduces the naturalness problem.**

- **Proton decay (and CLFV) -> R-parity conservation.**

- **R-parity conservation -> superpartners produced in pairs, and the LSP is stable.**

- **SUSY production cross sections at LHC could be large.**

- **Cross sections and kinematics depend on mass, spin, charges, and what other particles can be exchanged in the intitial state.**