The Violent Universe:
The Big Bang

Introduction to FRW Cosmology

The CMB

Big Bang Nucleosynthesis

- Comparison between Theory and Observations

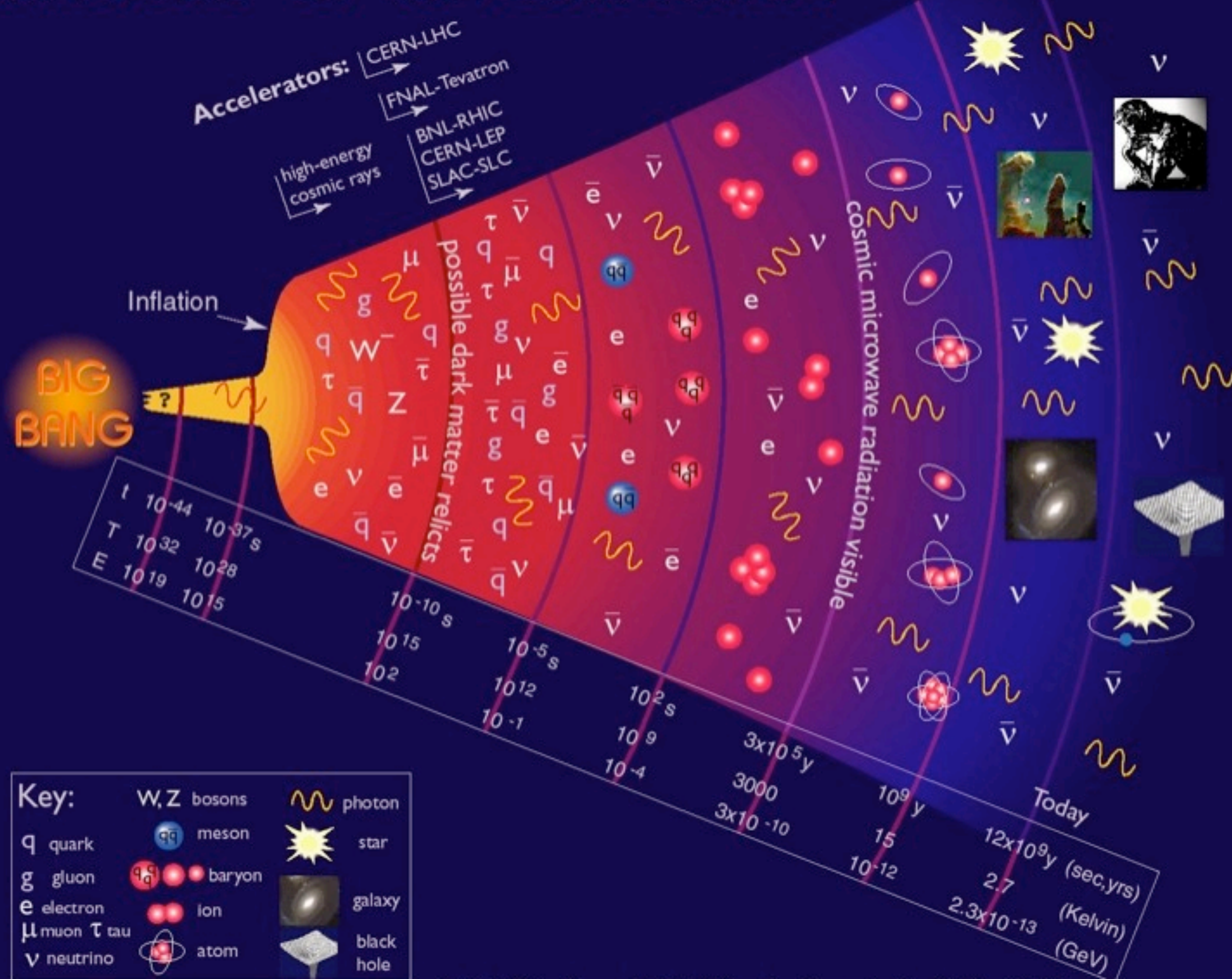
Dark Matter

- Evidence for Dark matter - Observation and theory
- Relic Densities
- Candidates beyond the Standard Model

Inflation

Baryogenesis

HISTORY OF THE UNIVERSE



Cosmological Principle

i) Copernican Principle: We are not privileged observers

ii) Relativity Principle: Physical Laws do not depend on space-time

→ There exists an infinite set of observers such that the universe is isotropic in all measurable properties at all times

→ The Universe is spatially homogeneous and isotropic

Consequences

i) The only true velocity fields can be expansion or contraction

velocity of observers depends only on spearation

$$v_{12} = H r_{12}$$

ii) The must exist a measure of distance independent of direction

$$d = z/H$$

Maximally symmetric spaces

Construction

$$ds^2 = C_{\mu\nu} dx^\mu dx^\nu + K^{-1} dz^2$$

with embedding

$$K C_{\mu\nu} x^\mu x^\nu + z^2 = 1$$

$$ds^2 = C_{\mu\nu} dx^\mu dx^\nu + \frac{K (C_{\mu\nu} x^\mu dx^\nu)^2}{(1 - K C_{\mu\nu} x^\mu x^\nu)}$$

Space-time with a maximally symmetric subspace

$$ds^2 = dt^2 - R^2(t) \left(d\vec{u}^2 + \frac{K (\vec{u} \cdot d\vec{u})^2}{(1 - K \vec{u}^2)} \right)$$

Friedmann-Robertson-Walker metric

$$ds^2 = dt^2 - R^2(t)\left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right)$$

R(t) is the scale factor

k is curvature constant :

k = -1, 0, +1 for spatially open, flat or closed Universes

with perfect-fluid source

$$T^{\mu\nu} = -pg^{\mu\nu} + (\rho + p)u^\mu u^\nu$$

and solve Einstein's equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}$$

The (00) component gives:

$$H^2 \equiv \frac{\dot{R}^2}{R^2} = \frac{8\pi G_N \rho}{3} - \frac{k}{R^2} + \frac{\Lambda}{3}$$

The (ii) components give:

$$\frac{\ddot{R}}{R} = \frac{\Lambda}{3} - \frac{4\pi G_N(\rho + 3p)}{3}$$

In addition $T^{\mu\nu}{}_{;\nu} = 0$ gives:

$$\dot{\rho} = -3H(\rho + p)$$

Consider k = Λ = 0

$$\frac{\dot{R}^2}{R^2} = \frac{8\pi G_N \rho}{3} \qquad \dot{\rho} = -3H(\rho + p)$$

i) Radiation dominated Universe: p = ϱ/3

ϱ ~ R^{-4} and R ~ $t^{1/2}$

ii) Matter dominated Universe: p = 0

ϱ ~ R^{-3} and R ~ $t^{2/3}$

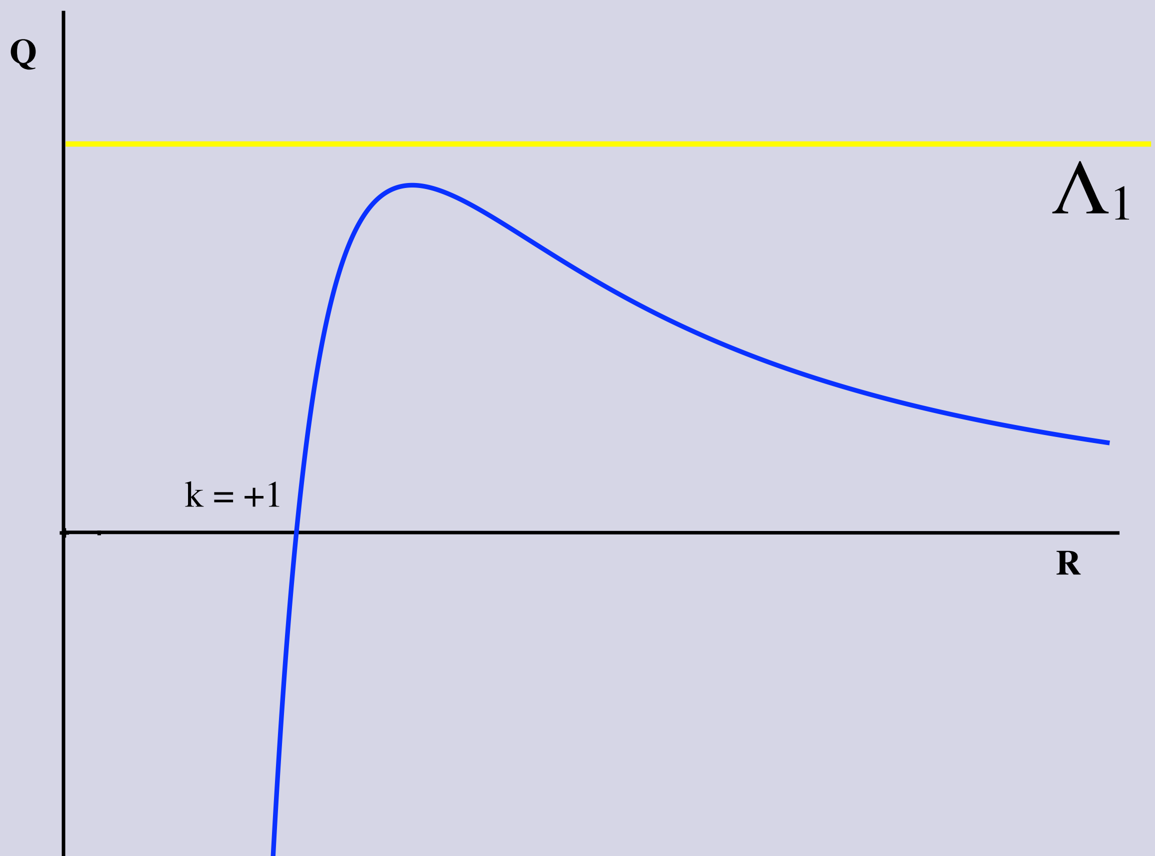
General Evolution

Define $Q = \frac{3k}{R^2} - 8\pi G_N \rho$, $p = (\gamma\text{-}1)\varrho$, $\varrho \sim R^{-3\gamma}$

$$\frac{\dot{R}}{R} = \left[\frac{\Lambda - Q}{3}\right]^{1/2} \Rightarrow \qquad Q \leq \Lambda$$

$k = +1$

$\Lambda_1 > Q_{max}$



i) Start at singularity at R = 0 and expand to R = ∞

ii) Start at R = ∞ and contract to singularity at R = 0

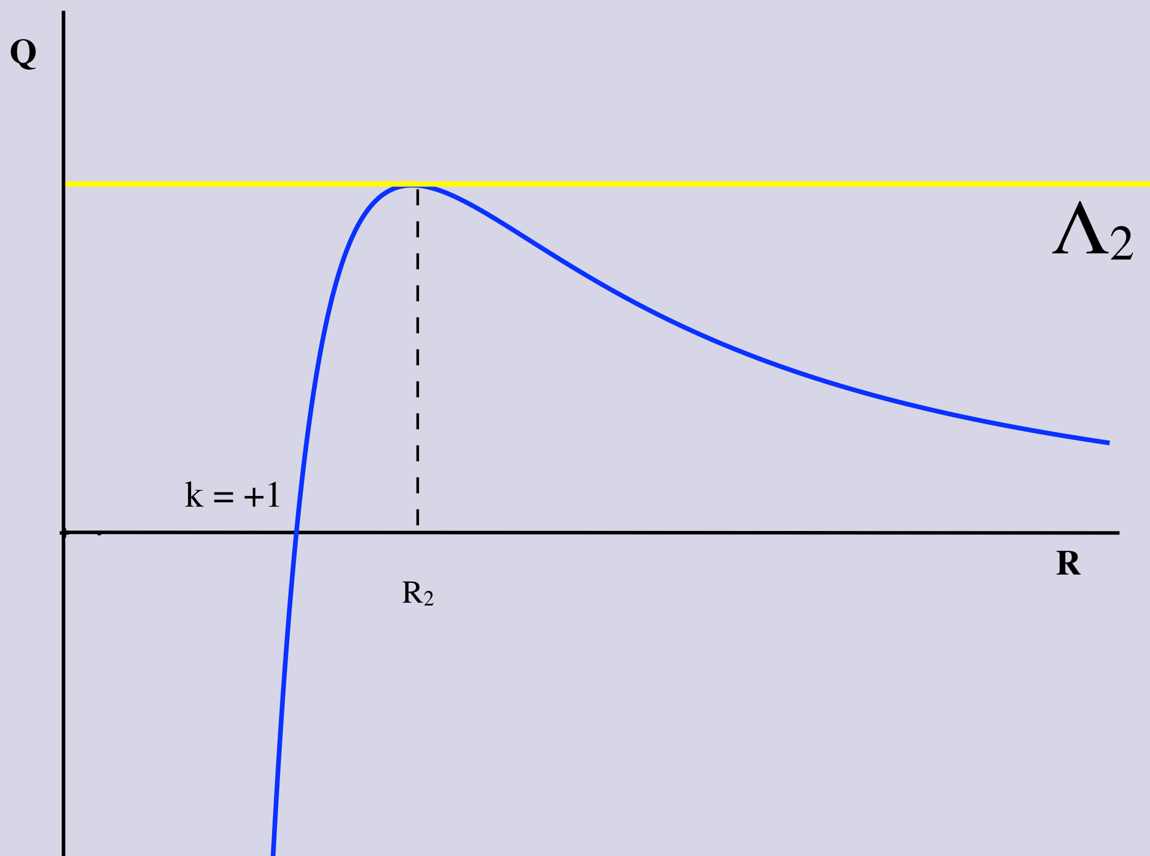
General Evolution

Define $Q = \frac{3k}{R^2} - 8\pi G_N \rho$, $p = (\gamma-1)\varrho$, $\varrho \sim R^{-3\gamma}$

$$\frac{\dot{R}}{R} = \left[\frac{\Lambda - Q}{3}\right]^{1/2} \Rightarrow \qquad Q \leq \Lambda$$

$k = +1$

$\Lambda_2 = Q_{max}$



i) Start at singularity at $R = 0$ and expand to $R = R_2$

ii) Start near $R = R_2$ and contract to singularity at $R = 0$

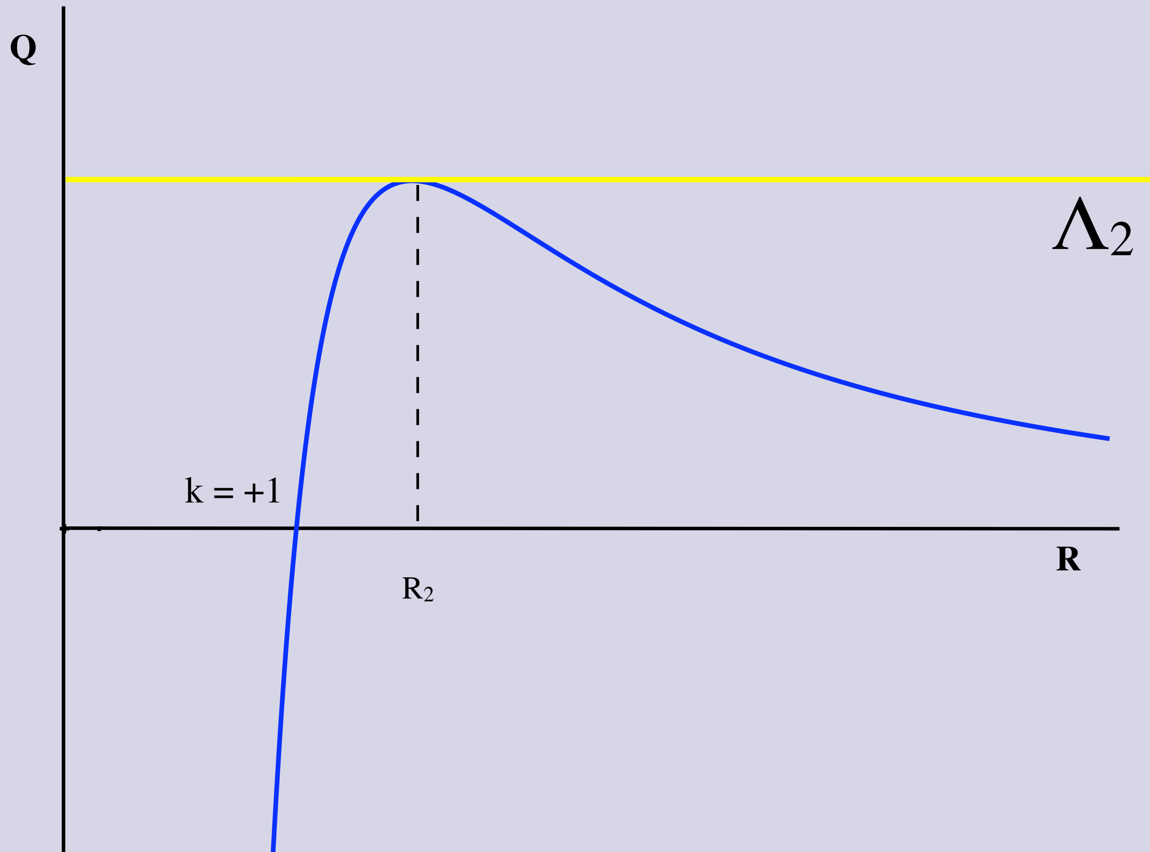
General Evolution

Define $Q = \frac{3k}{R^2} - 8\pi G_N \rho$, $p = (\gamma\text{-}1)\varrho$, $\varrho \sim R^{-3\gamma}$

$$\frac{\dot{R}}{R} = \left[\frac{\Lambda - Q}{3}\right]^{1/2} \Rightarrow \qquad Q \leq \Lambda$$

k = +1

$\Lambda_2 = Q_{max}$



ii) Start near $R = R_2$ and contract to singularity at $R = 0$
iii) Start near $R = R_2$ and expand to $R = \infty$
iv) Start at $R = \infty$ and contract to $R = R_2$
v) Einstein Static Univ.

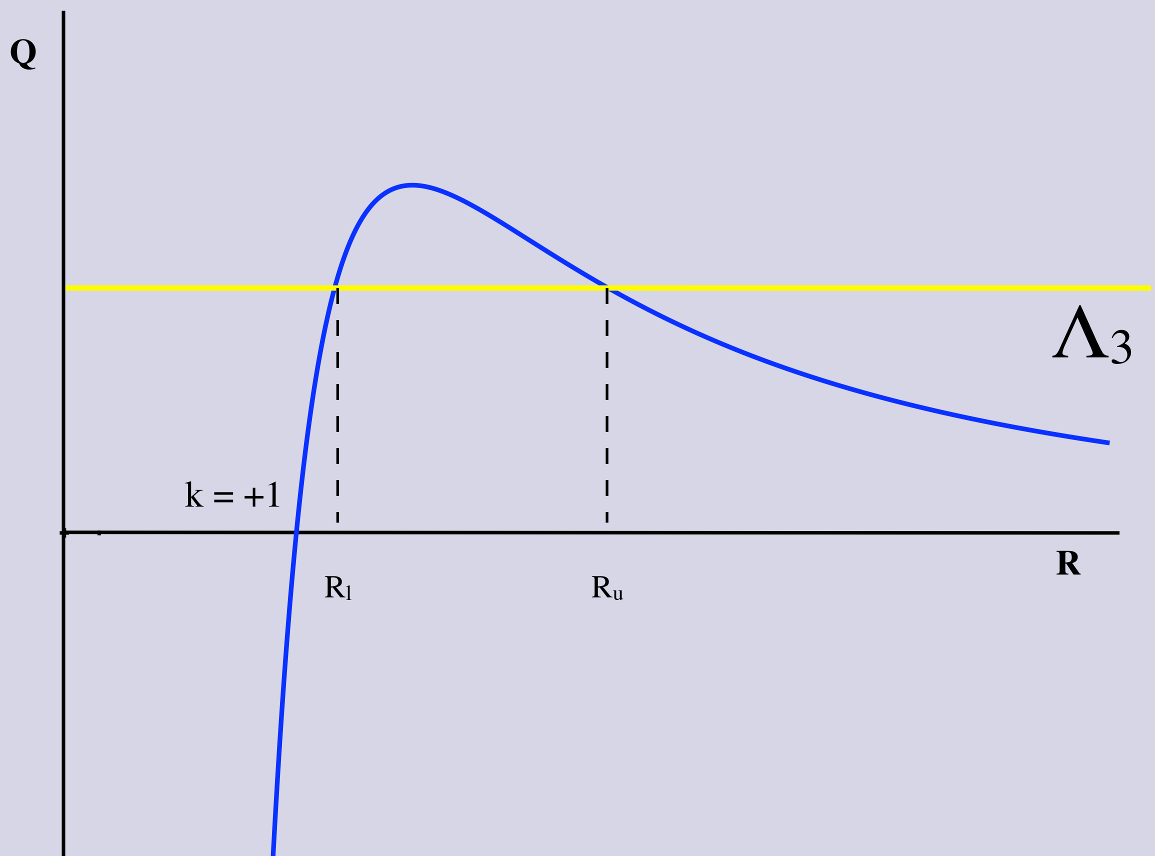
General Evolution

Define $Q = \frac{3k}{R^2} - 8\pi G_N \rho$, p = (γ-1)ϱ, ϱ ~ $R^{-3\gamma}$

$$\frac{\dot{R}}{R} = \left[\frac{\Lambda - Q}{3}\right]^{1/2} \Rightarrow \qquad Q \leq \Lambda$$

k = +1

$0 < \Lambda_3 < Q_{max}$



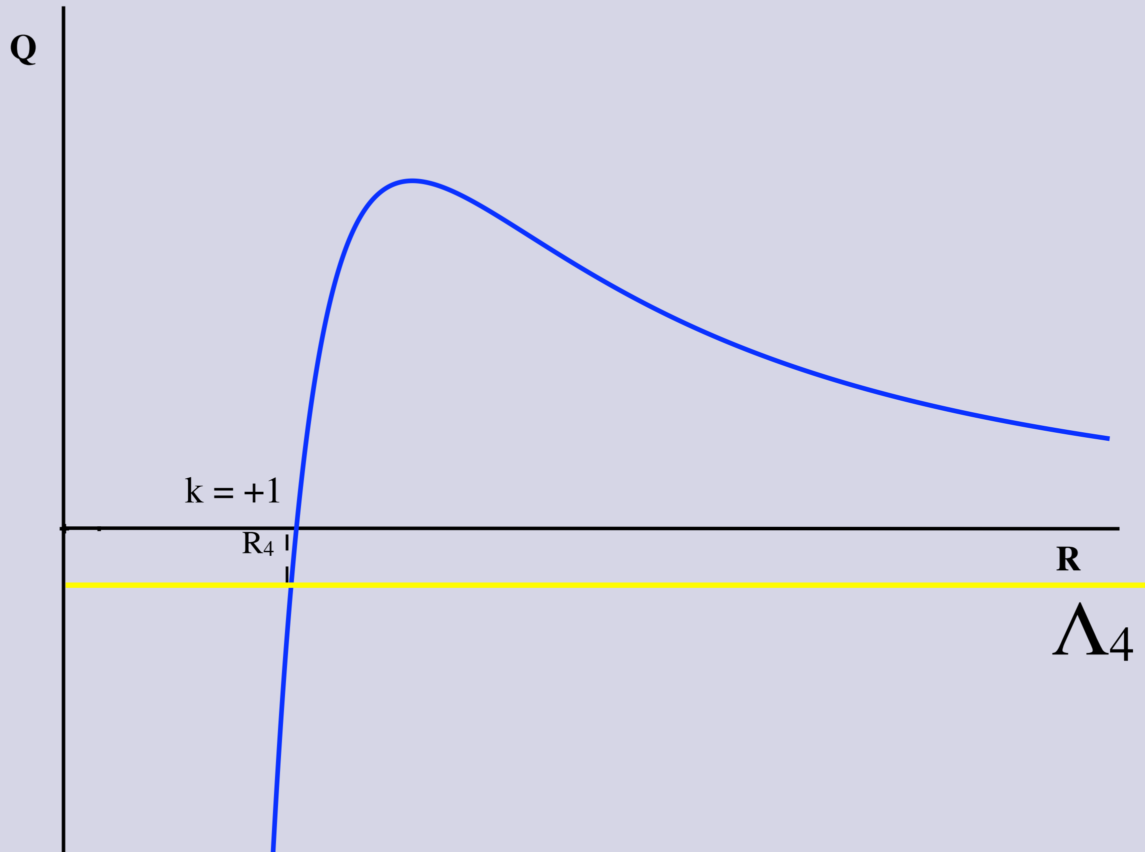
i) Start at singularity at R = 0 and expand to R = R_l and recollapse to R = 0

ii) Start at R = ∞ and contract to R = R_u and then reexpand to R = ∞

General Evolution

Define $\boldsymbol{Q = \frac{3k}{R^2} - 8\pi G_N \rho}$, $p = (\gamma\text{-}1)\varrho$, $\varrho \sim R^{-3\gamma}$

$$\frac{\dot{R}}{R} = \left[\frac{\Lambda - Q}{3}\right]^{1/2} \Rightarrow \qquad Q \leq \Lambda$$



$k = +1$

$\Lambda_4 \leq 0$

i) Start at singularity at $R = 0$ and expand to $R = R_4$ and recollapse to $R = 0$

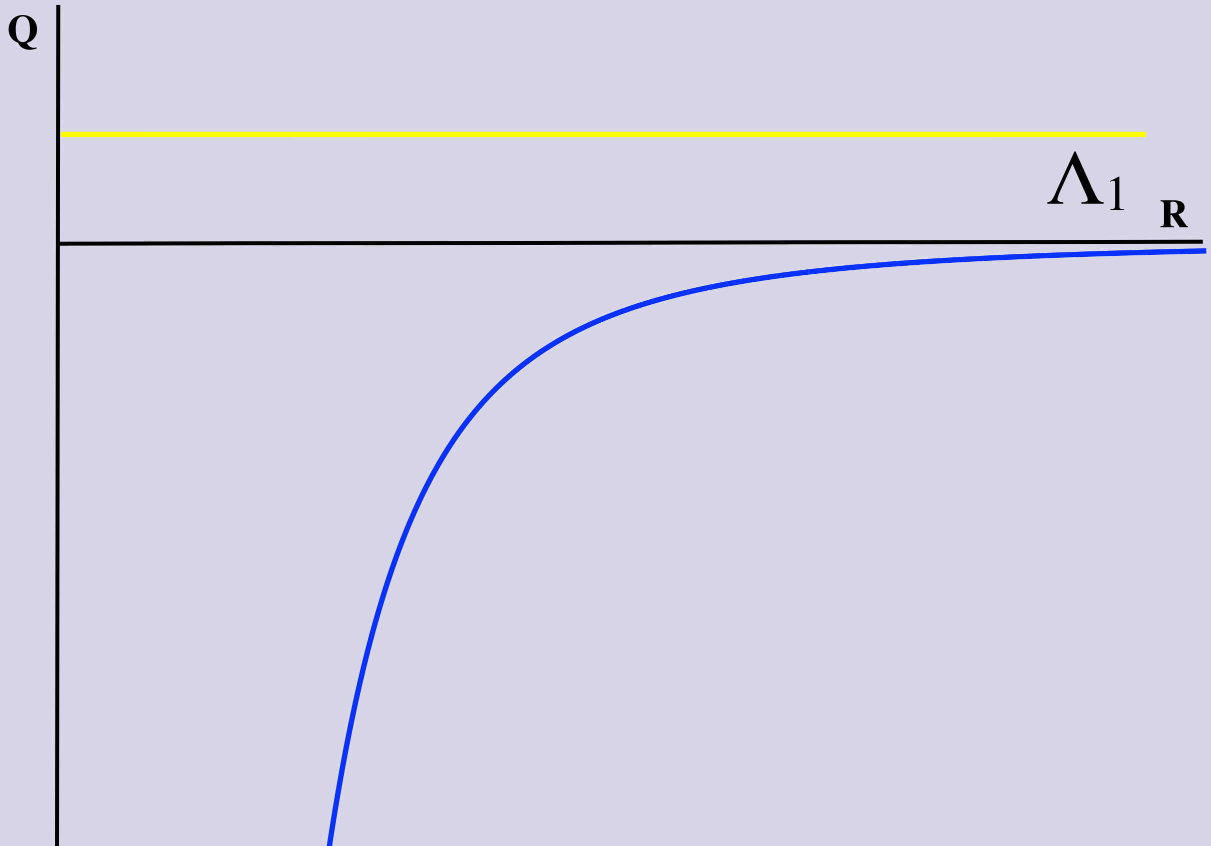
General Evolution

Define $Q = \dfrac{3k}{R^2} - 8\pi G_N \rho$, p = (γ-1)ϱ, $\varrho \sim R^{-3\gamma}$

$$\frac{\dot{R}}{R} = \left[\frac{\Lambda - Q}{3}\right]^{1/2} \Rightarrow \qquad Q \leq \Lambda$$

k = 0, -1

$\Lambda_1 \geq 0$



i) Start at singularity at R = 0 and expand to R = ∞

ii) Start at R = ∞ and contract to singularity at R = 0

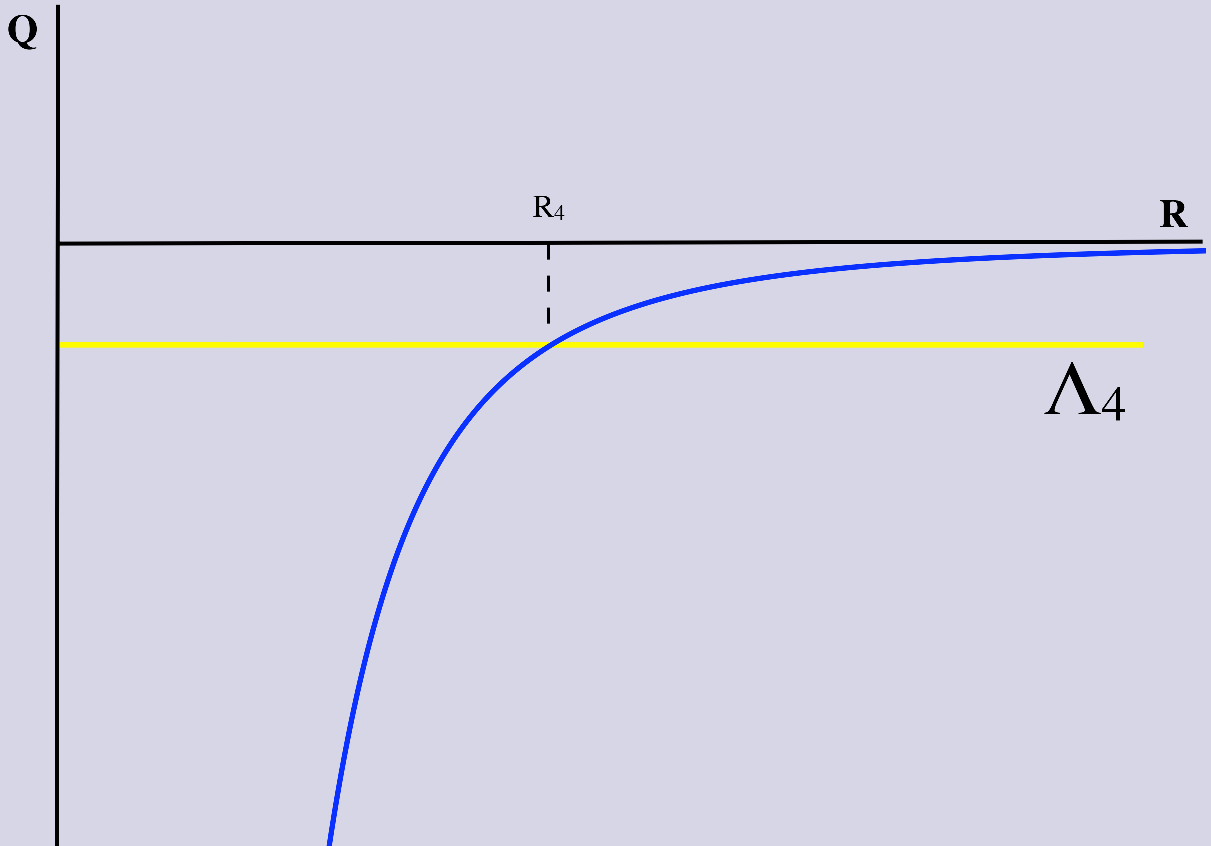
General Evolution

Define $Q = \frac{3k}{R^2} - 8\pi G_N \rho$, $p = (\gamma\text{-}1)\varrho$, $\varrho \sim R^{-3\gamma}$

$$\frac{\dot{R}}{R} = \left[\frac{\Lambda - Q}{3}\right]^{1/2} \Rightarrow \qquad Q \leq \Lambda$$

$k = 0, -1$

$\Lambda_4 < 0$



i) Start at singularity at $R = 0$ and expand to $R = R_4$ and recollapse to $R = 0$

Conformal Coordinates

$$ds^2 = dt^2 - R^2(t)\left(d\chi^2 + f^2(\chi)(d\theta^2 + \sin^2\theta d\phi^2)\right)$$

$$f(\chi) = \begin{cases} \sinh\chi & k = -1 \\ \chi & k = 0 \\ \sin\chi & k = +1 \end{cases}$$

Conformal time

$$R d\eta = dt$$

$$ds^2 = R^2(\eta)\left[d\eta^2 - d\chi^2 - f^2(\chi)(d\theta^2 + \sin^2\theta d\phi^2)\right]$$

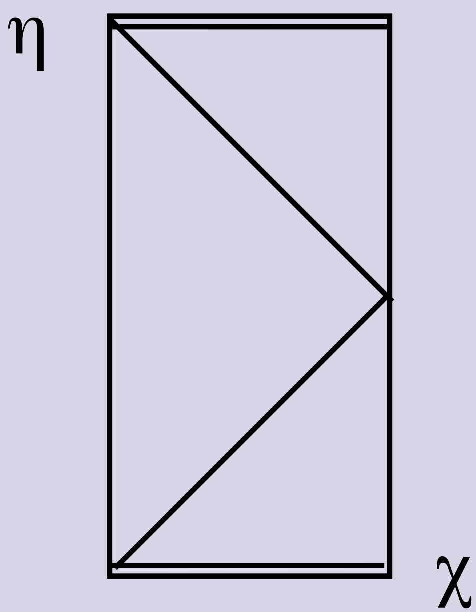
Solutions

Friedman equation becomes

$$R'' + kR = \frac{4\pi G_N}{3}(\rho - 3p)R^3$$

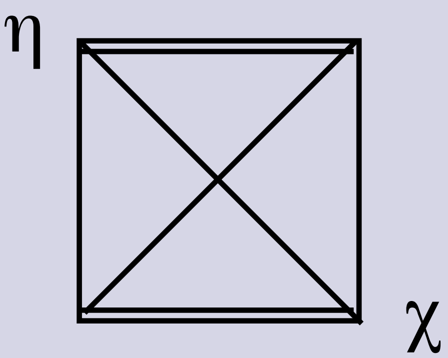
$$R = \begin{cases} \cosh\eta - 1 & k = -1 \\ \eta^2/2 & k = 0 \\ 1 - \cos\eta & k = +1 \end{cases} \qquad t = \begin{cases} \sinh\eta - \eta & k = -1 \\ \eta^3/6 & k = 0 \\ \eta - \sin\eta & k = +1 \end{cases}$$

p = 0



$$R = \begin{cases} \sinh\eta & k=-1 \\ \eta & k=0 \\ \sin\eta & k=+1 \end{cases} \qquad t = \begin{cases} \cosh\eta - 1 & k=-1 \\ \eta^2/2 & k=0 \\ 1-\cos\eta & k=+1 \end{cases}$$

$p = \varrho/3$



The Universe today

Define the deceleration parameter : $$q_0 = -\frac{\ddot{R}_0 R_0}{\dot{R}_0^2}$$

$$\frac{\ddot{R}}{R} = \frac{\Lambda}{3} - \frac{4\pi G_N(\rho + 3p)}{3}$$ becomes (with p << ϱ)

$$-2q_0 H_0^2 = \frac{2\Lambda}{3} - \frac{8\pi G_N \rho_0}{3} = \Lambda - H_0^2 - \frac{k}{R_0^2}$$

or $$\frac{k}{R_0^2} = \Lambda + H_0^2(2q_0 - 1)$$

or $$\frac{k}{R_0^2} = H_0^2(\frac{3}{2}\Omega_0 - q_0 - 1)$$

where $\Omega = \rho/\rho_c$ $\quad \rho_c = 3H^2/8\pi G_N = 1.88 \times 10^{-29} h^2$ g cm^{-3}

and h = H/100 (km/Mpc/s)

The Universe today

When $\Lambda = 0$,

$$q_0 = \frac{4\pi G_N \rho_0}{3H_0^2} = \frac{\Omega_0}{2}$$

and

$$\frac{k}{R_0^2} = H_0^2(\Omega_0 - 1)$$

(still true for $p \neq 0$)

Evolution of Ω

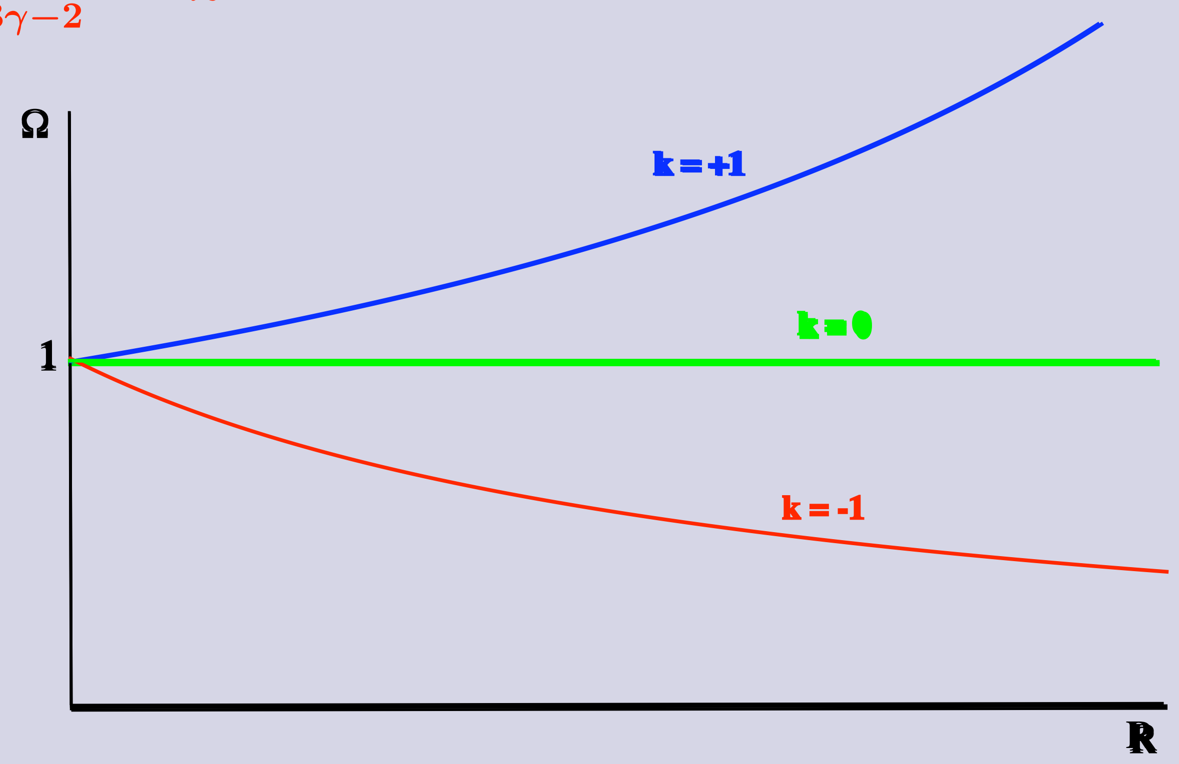
$(\Lambda = 0)$

$$\Omega = \frac{k}{R^2 H^2} + 1$$

$k = 0 \Rightarrow \Omega = 1$ always

write $$R^2 H^2 = \frac{8\pi G_N A}{3R^{3\gamma-2}} - k$$

$$\Omega = \frac{k}{\frac{8\pi G_N A}{3R^{3\gamma-2}} - k} + 1$$



Age of the Universe

$(\Lambda = 0)$

$$\frac{\dot{R}^2}{R^2} = \frac{8\pi G_N \rho}{3} - \frac{k}{R^2}$$

$$\rho = \rho_0 \left(\frac{R_0}{R}\right)^{3\gamma} \qquad \frac{k}{R_0^2} = H_0^2(\Omega_0 - 1) \qquad x = R/R_0$$

$$\dot{x}^2 = \Omega_0 H_0^2 (\frac{R_0}{R})^{3\gamma-2} - (\Omega_0 - 1) H_0^2$$

$$\dot{x} = H_0 \left[1 - \Omega_0 + \Omega_0 x^{2-3\gamma}\right]^{1/2}$$

$$H_0 t = \int_0^1 \frac{dx}{\left[1 - \Omega_0 + \Omega_0 x^{2-3\gamma}\right]^{1/2}}$$

Age of the Universe

$(\Lambda = 0)$

Special cases: $\Omega_0 = 1$

$\gamma = 1$

$$t = \frac{2}{3H}$$

$\gamma = 4/3$

$$t = \frac{1}{2H}$$

Proper Distance

$$d_p = R(t) \int_0^{r_1} \frac{dr'}{\sqrt{1 - kr'^2}}$$

For light paths,

$$\int_{t_1}^{t} \frac{dt'}{R(t')} = \int_0^{r_1} \frac{dr'}{\sqrt{1 - kr'^2}}$$

as $t_1 \rightarrow 0$, r_1 is the maximum distance from which we can receive a signal

Particle Horizon

$$d_H = R(t) \int_0^{r_H} \frac{dr'}{\sqrt{1 - kr'^2}} = R(t) \int_0^{t} \frac{dt'}{R(t')}$$

Proper Distance

$(\Lambda = 0)$

Special cases: $\Omega_0 = 1$

$\gamma = 1$

$$\boldsymbol{d_H = 3t}$$

$\gamma = 4/3$

$$\boldsymbol{d_H = 2t}$$

d_H/R grows with time $\Rightarrow$ we see more of the universe as time goes on

Redshift

$$z \equiv \frac{\nu_e - \nu_o}{\nu_o} = v_{rel}$$

If s = R δr

$$v = \dot{s} = \dot{R}\delta r = \frac{\dot{R}}{R} R\delta r = H s$$

$$1 + z = \frac{\nu_e}{\nu_o} = 1 + \frac{\dot{R}\delta t}{R} = \frac{R_o}{R}$$

Angular Size vs redshift

Proper diameter $D = R(t_e)\, r\, \delta = s\, \delta$

$$\boldsymbol{s = R(t_e) \int_{t_e}^{t_o} \frac{dt'}{R(t')}}$$

For $k = 0$, $\gamma = 1$

$$\boldsymbol{s = \frac{2}{H_0(1+z)} \left[1 - (1+z)^{-1/2}\right]}$$

small z, $\delta \sim HD/z$

large z, $\delta \sim HDz/2$

Redshift-Magnitude

Luminosity distance

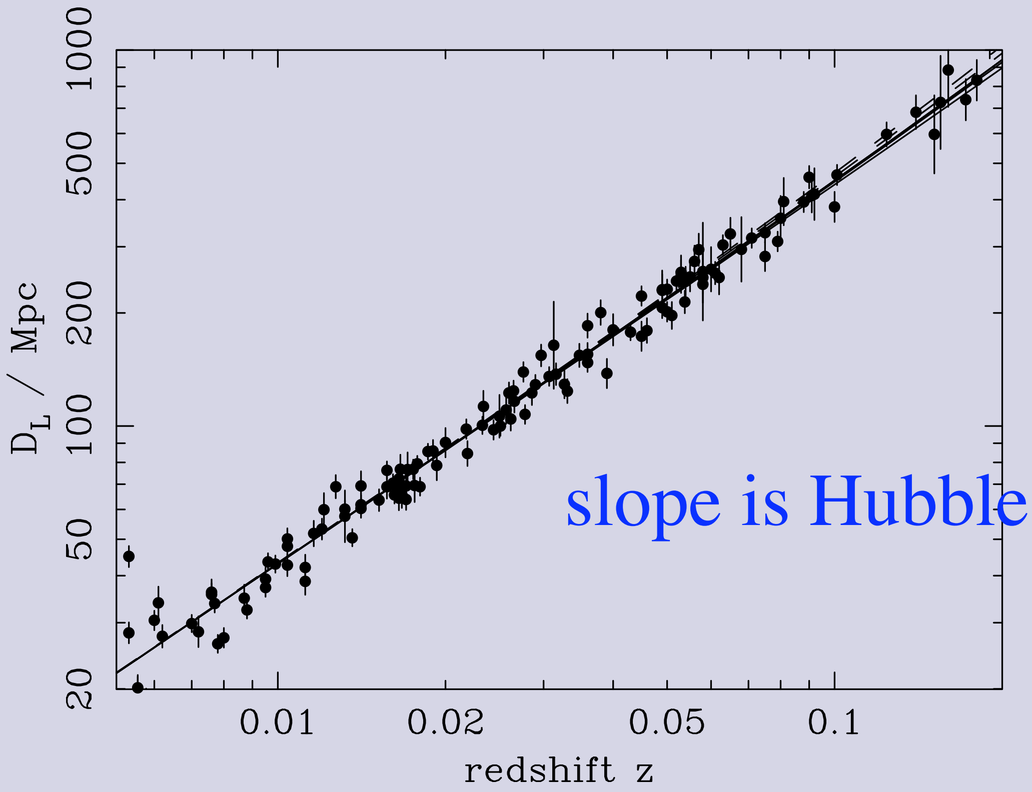
Observed Flux $$F = \frac{L_e}{4\pi d_L^2} = \frac{L_o}{4\pi (R_0 r)^2}$$

so, $$d_L^2 = \frac{L_e}{L_o}(R_0 r)^2 \qquad d_L = (1+z)R_0 r = (1+z)^2 s$$

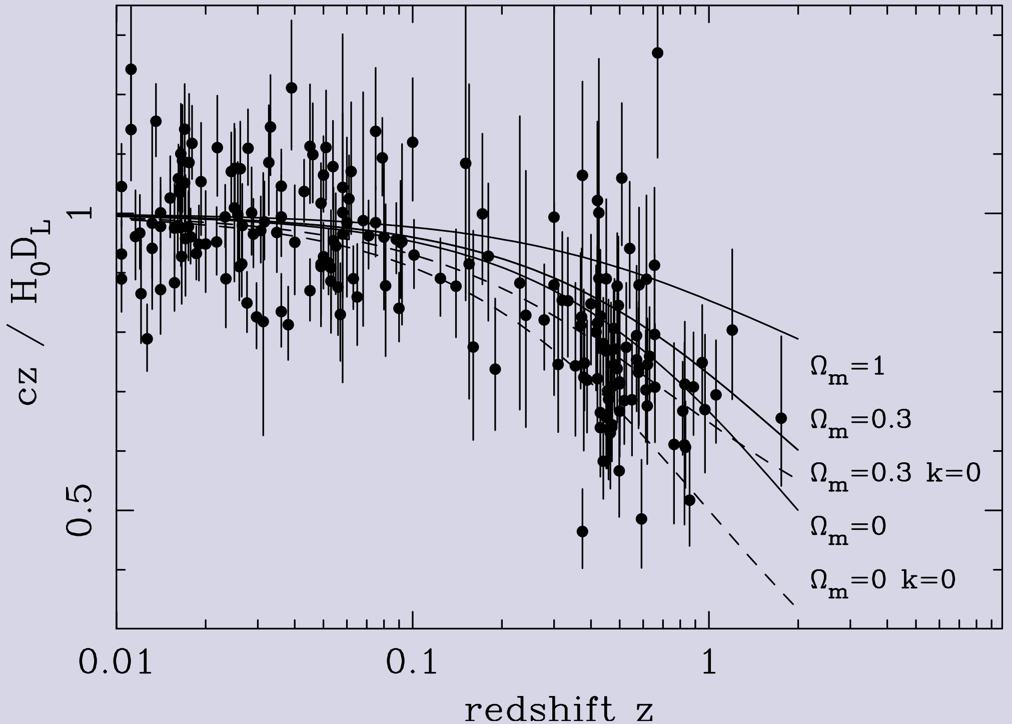
Define $$m = -2.5\log F \qquad M = -2.5\log Le$$

$$m - M = -5 + 5\log d_L/pc$$

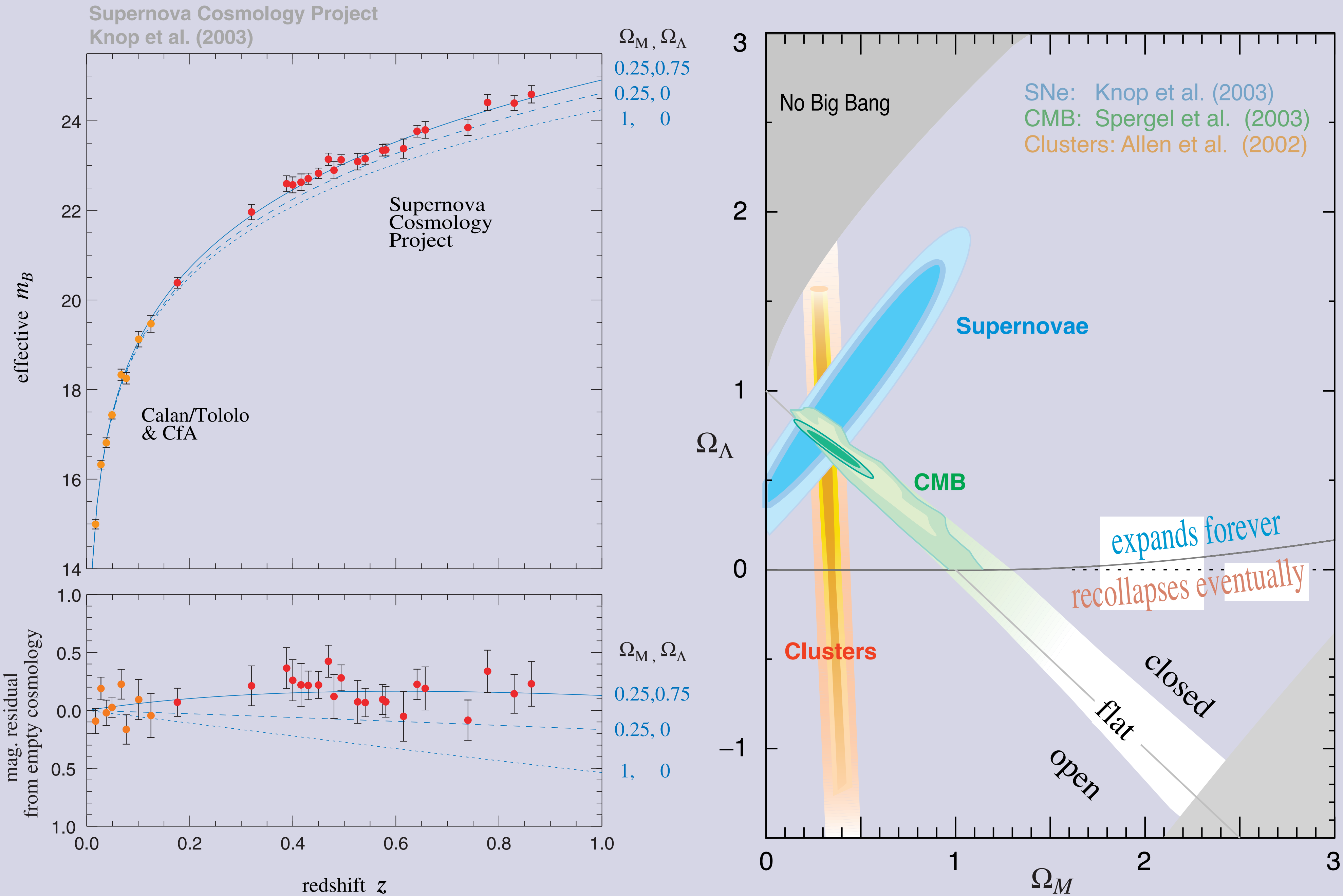
$$= -5 - \log H + 5\log z + 5\log(1 + (1-q_0)z/2)$$



slope is Hubble law



Perlemutter et al.
Riess et al.
Tonry et al.



Supernova Cosmology Project
Knop et al. (2003)
effective m_B
Calan/Tololo & CfA
Supernova Cosmology Project
Ω_M, Ω_Λ
0.25,0.75
0.25, 0
1, 0
mag. residual from empty cosmology
redshift z
No Big Bang
SNe: Knop et al. (2003)
CMB: Spergel et al. (2003)
Clusters: Allen et al. (2002)
Supernovae
CMB
Clusters
expands forever
recollapses eventually
closed
flat
open
Ω_Λ
Ω_M

The Hot Thermal Universe

The energy density in photons:

$$\rho_\gamma = \int E_\gamma dn_\gamma$$

with density of states ($g_\gamma = 2$)

$$dn_\gamma = \frac{g_\gamma}{2\pi^2}[exp(E_\gamma/T) - 1]^{-1} q^2 dq$$

giving

$$\rho_\gamma = \frac{\pi^2}{15}T^4 \quad p_\gamma = \frac{1}{3}\rho_\gamma \quad s_\gamma = \frac{4\rho_\gamma}{3T} \quad n_\gamma = \frac{2\zeta(3)}{\pi^2}T^3$$

In general,

$$\rho_i = \int E_i dn_{q_i}$$

with

$$dn_{q_i} = \frac{g_i}{2\pi^2}[exp[(E_{q_i} - \mu_i)/T] \pm 1]^{-1} q^2 dq$$

and

$$E_{q_i} = \left(m_i^2 + q_i^2\right)^{1/2}$$

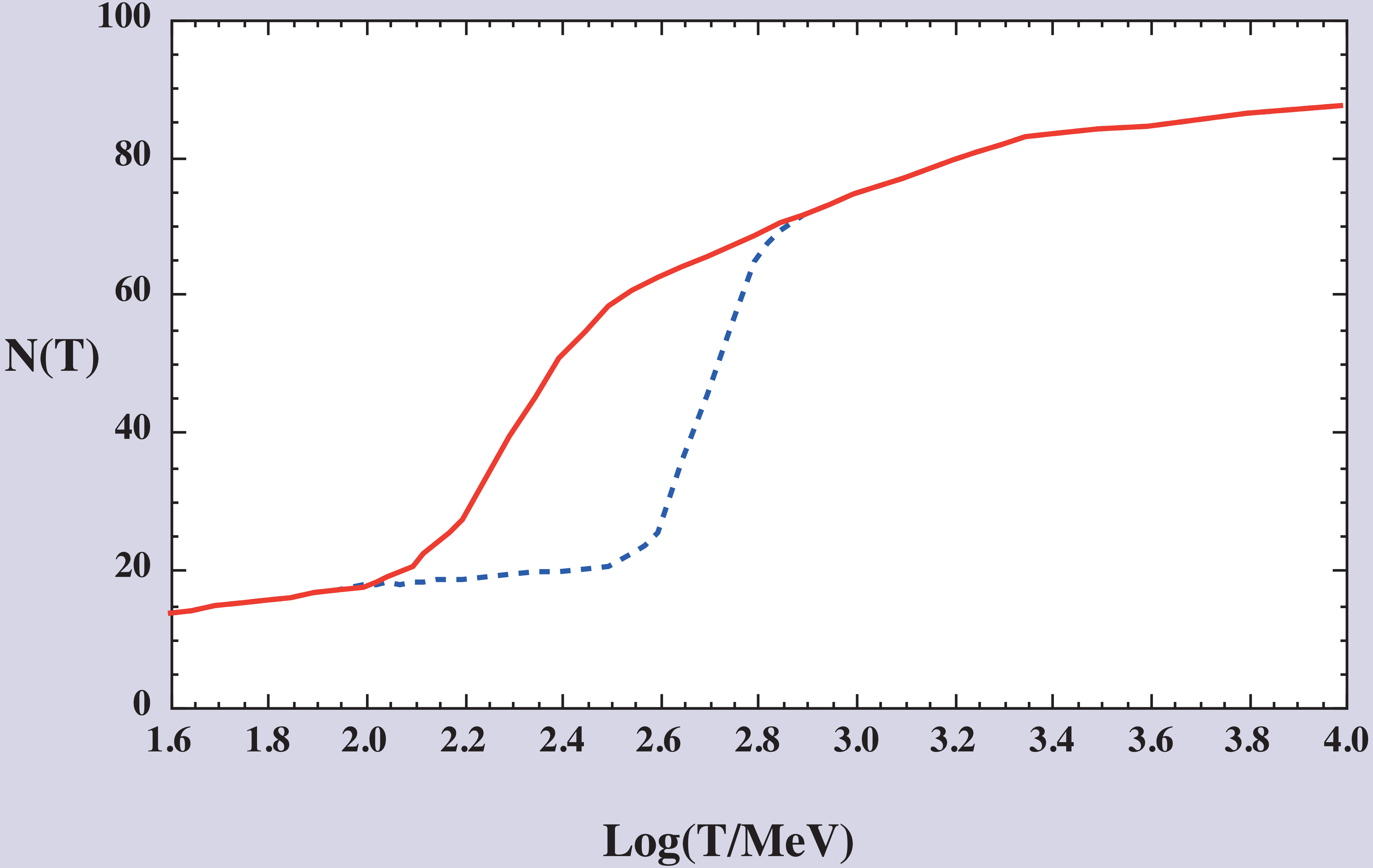
For Radiation $m_i << T$:

$$\rho = \left(\sum_B g_B + \frac{7}{8} \sum_F g_F \right) \frac{\pi^2}{30} T^4 \equiv \frac{\pi^2}{30} N(T)\, T^4$$

Table 1: Effective numbers of degrees of freedom in the standard model.

Temperature	New Particles	$4N(T)$
$T < m_e$	γ's + ν's	29
$m_e < T < m_\mu$	$e^\pm$	43
$m_\mu < T < m_\pi$	$\mu^\pm$	57
$m_\pi < T < Tc^*$	π's	69
$T_c < T < m_{\text{strange}}$	- π's + $u, \bar{u}, d, \bar{d}$ + gluons	205
$m_s < T < m_{charm}$	$s, \bar{s}$	247
$m_c < T < m_\tau$	$c, \bar{c}$	289
$m_\tau < T < m_{bottom}$	$\tau^\pm$	303
$m_b < T < m_{W,Z}$	$b, \bar{b}$	345
$m_{W,Z} < T < m_{top}$	$W^\pm, Z$	381
$m_t < T < m_{Higgs}$	$t, \bar{t}$	423
$M_H < T$	H^o	427

*T_c corresponds to the confinement-deconfinement transition between quarks and hadrons. $N(T)$ is shown in Figure 1 for $T_c = 150$ and 400 MeV. It has been assumed that $m_{Higgs} > m_{top}$.



100
80
60
40
20
0
N(T)
1.6
1.8
2.0
2.2
2.4
2.6
2.8
3.0
3.2
3.4
3.6
3.8
4.0
Log(T/MeV)

Time-temperature Relation

Recall $\gamma = 4/3$

$$t = \frac{1}{2H} \qquad H^2 = \frac{8\pi G_N \rho}{3}$$

$$t = \left(\frac{3}{32\pi G_N \rho}\right)^{1/2} = \left(\frac{90}{32\pi^3 G_N N(T)}\right)^{1/2} T^{-2}$$

or

$$t_s T_{MeV}^2 = \frac{2.41}{\sqrt{N(t)}}$$

Equilibrium

- Particles will be in equilibrium if there is a reaction rate which is fast enough: $\Gamma > H$

Neutrinos

kept in thermal equilibrium by processes such as

$$e^+ + e^- \leftrightarrow \nu + \bar{\nu} \text{ or } e + \nu \leftrightarrow e + \nu \text{ etc.,}$$

with $\Gamma = n\langle\sigma v\rangle$ and $\langle\sigma v\rangle \sim 0(10^{-2})T^2/M_W^4$

$$\Gamma_{wk} \sim 0(10^{-2})T^5/M_W^4$$

Neutrinos in equilibrium when

$$T > (500M_W^4)/M_P)^{1/3} \sim 1MeV.$$

Entropy Conservation

Energy conservation: $T^{\mu\nu}{}_{;\nu} = 0$

$$\dot{\rho} = -3H(\rho + p)$$

equivalent to

$$\dot{p}R^3 = \frac{d}{dt}(R^3(\rho + p)) \quad = \frac{d}{dt}(R^3 T s)$$

Now,
$$\dot{p} = \frac{dp}{dT}\frac{dT}{dt} = s\frac{dT}{dt}$$

so
$$s\frac{dT}{dt}R^3 = \frac{d}{dt}(R^3 T s) = s\frac{dT}{dt}R^3 + T\frac{d}{dt}(R^3 s)$$

$$\Longrightarrow \qquad \frac{d}{dt}(R^3 s) = 0$$

Neutrino Temperature

- At T ~ 1 MeV neutrinos decouple
- At T ~ 1/2 MeV e^+ e^- annihilate to photons
- Entropy of "γ's" and ν's conserved speparately
- Prior to annihilation, $T_\gamma = T_\nu = T_>$

$$s_> = \frac{4}{3}\frac{\rho_>}{T_>} = \frac{4}{3}(2 + \frac{7}{2})(\frac{\pi^2}{30})T_>^3$$

- After annihilation, $T_\gamma = T_<$ but, $T_\nu = T_>$

$$s_< = \frac{4}{3}\frac{\rho_<}{T_<} = \frac{4}{3}(2)(\frac{\pi^2}{30})T_<^3$$

$$T_\nu = (4/11)^{1/3} T_\gamma \simeq 1.9K$$

The CMB

Historical Perspective

Intimate connection with CMB

Alpher
Herman
Gamow

Conditions for BBN:

Require T > 100 keV $\Rightarrow$ t < 200 s

$$\sigma v(p + n \rightarrow D + \gamma) \approx 5 \times 10^{-20}\ \mathrm{cm}^3/\mathrm{s}$$

$$\Rightarrow n_B \sim 1/\sigma v t \sim 10^{17}\ \mathrm{cm}^{-3}$$

Today:

$$n_{Bo} \sim 10^{-7}\ \mathrm{cm}^{-3}$$

and

$$n_B \sim R^{-3} \sim T^3$$

Predicts the CMB temperature

$$T_o = (n_{Bo} / n_B)^{1/3}\, T_{BBN} \sim 10\ \mathrm{K}$$

Some History:

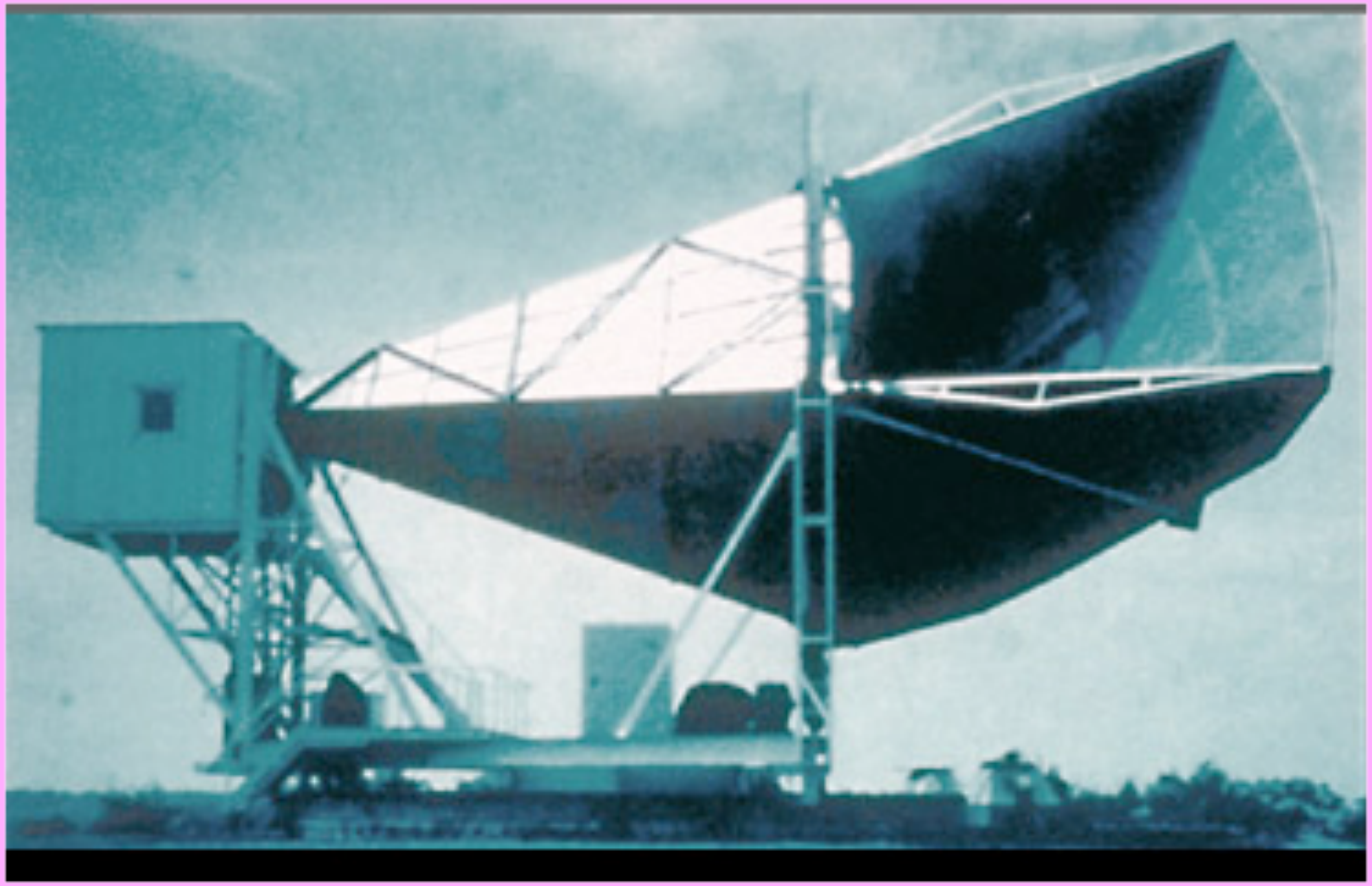
Penzias and Wilson:
Perfecting a radio antenna to track the Echo satellite
found background noise which could not be eliminated.

Corresponding temperature:

$T = 3.5 \pm 1$ K

Published in
"A Measurement of Excess Antenna Temperature at 4080 Hz"

Followed by an explanation by Dicke, Peebles Roll, & Wilkenson



Subsequently, many measurements (ground and balloon based) showed that:

$$T = 2.7 - 3 \text{ K}$$

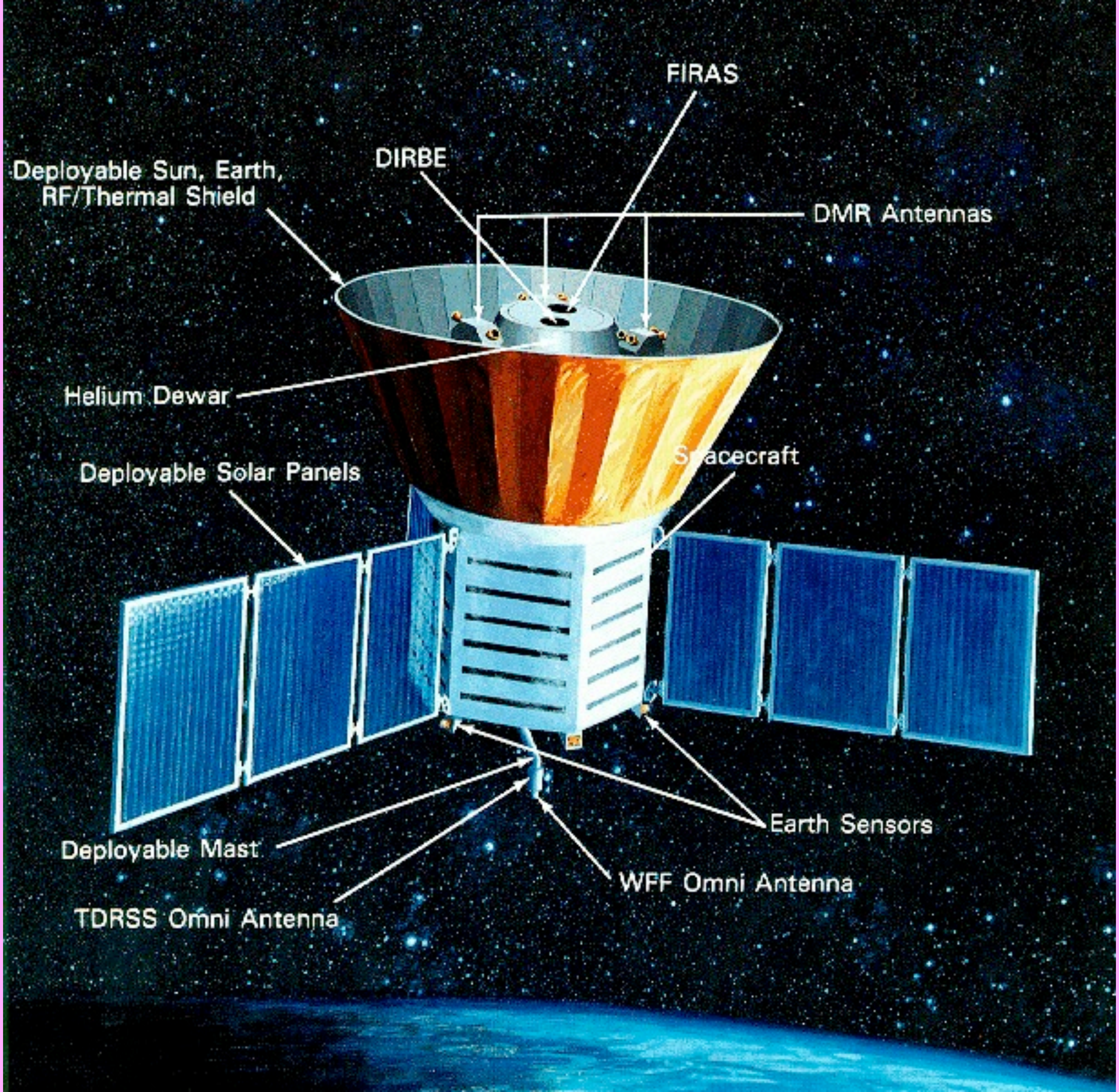
Enter COBE.

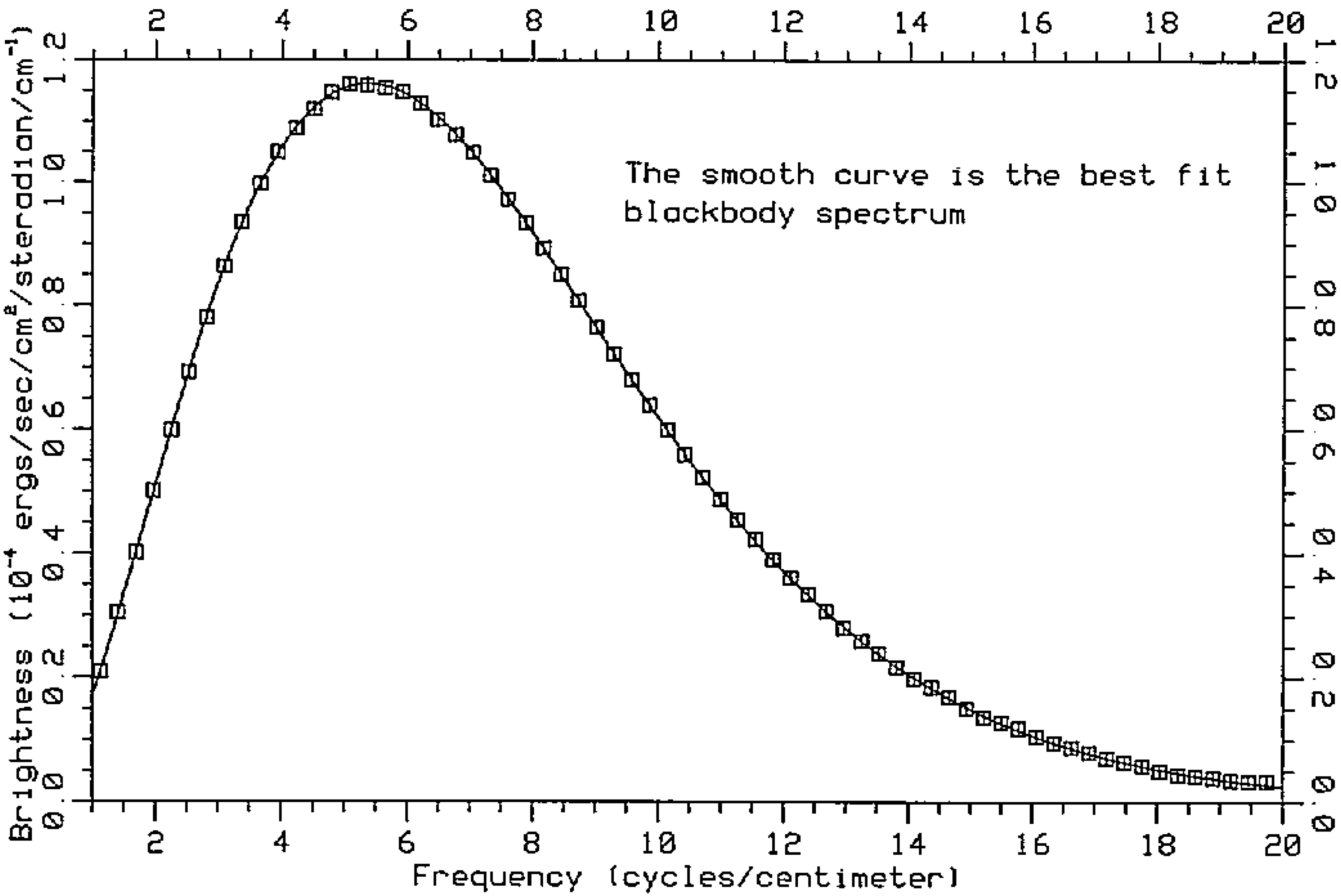
Lingering doubts regarding distortions and aniotropies set aside.

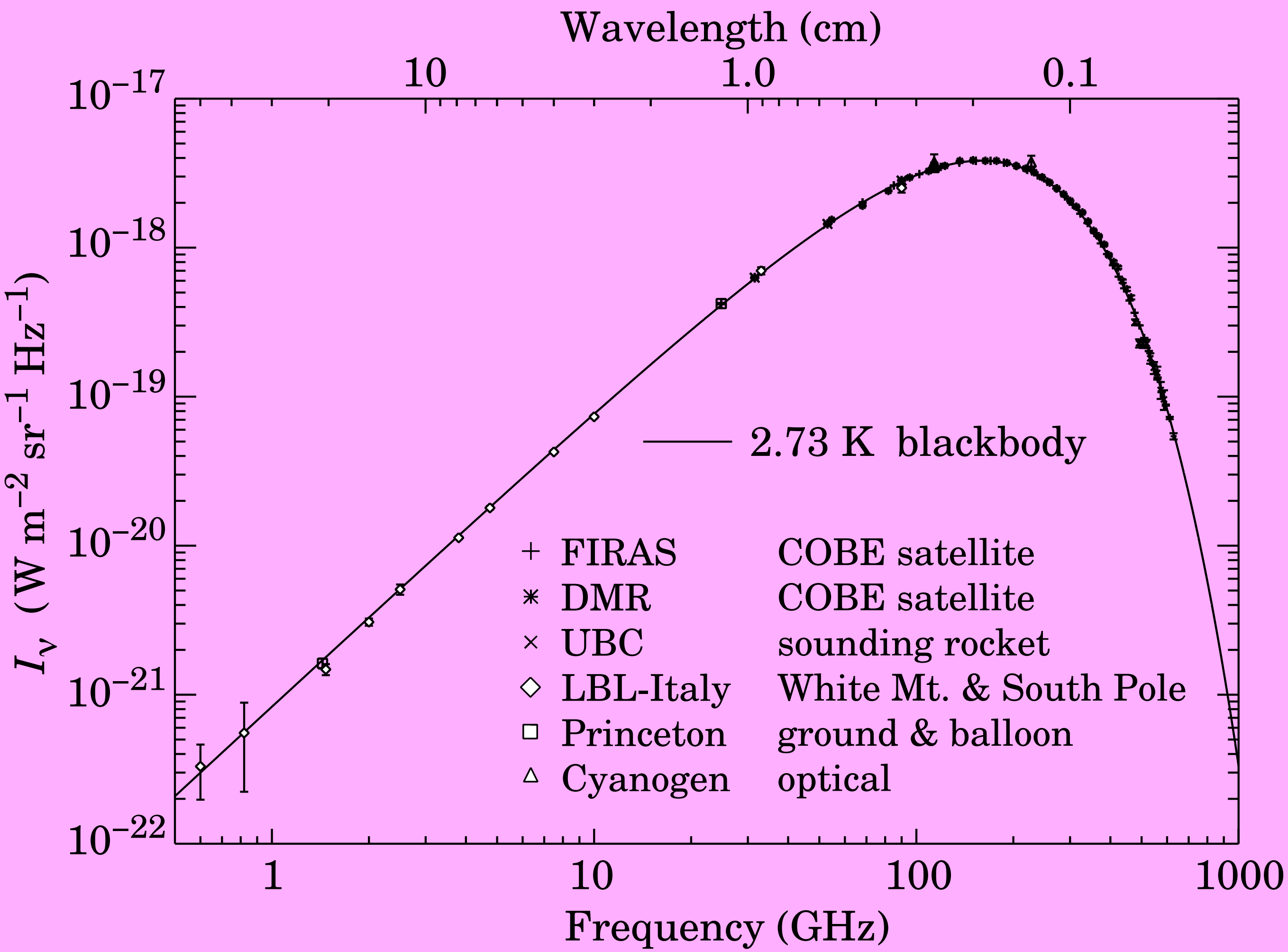
$$T = 2.73 \pm 0.01 \text{ K}$$

$$n_\gamma \sim T^3 = 411 \text{ cm}^{-3}$$

$$\rho_\gamma \sim T^4 \leq 10^{-4} \rho_c$$


FIRAS
DIRBE
Deployable Sun, Earth,
RF/Thermal Shield
DMR Antennas
Helium Dewar
Spacecraft
Deployable Solar Panels
Earth Sensors
Deployable Mast
WFF Omni Antenna
TDRSS Omni Antenna


The smooth curve is the best fit blackbody spectrum
Brightness (10^{-4} ergs/sec/cm^2/steradian/cm^{-1})
Frequency (cycles/centimeter)
2
4
6
8
10
12
14
16
18
20
0.0
0.2
0.4
0.6
0.8
1.0
1.2


Wavelength (cm)
10
1.0
0.1
10^{-17}
10^{-18}
10^{-19}
10^{-20}
10^{-21}
10^{-22}
I_ν (W m^{-2} sr^{-1} Hz^{-1})
2.73 K blackbody
FIRAS COBE satellite
DMR COBE satellite
UBC sounding rocket
LBL-Italy White Mt. & South Pole
Princeton ground & balloon
Cyanogen optical
1
10
100
1000
Frequency (GHz)

COBE DMR Microwave Sky at 53 GHz

Anisotropies

- The Universe is NOT completely homogeneous and isotropic
  - If $\langle \varrho \rangle \sim 0.1\ \varrho_c = 10^{-30}$ g cm^{-3}
  - In our Galaxy, $M = 10^{11} M_\odot = 10^{44}$ gm in a volume π (30 kpc)^2 300 pc = 10^{67} cm^3
    - $\varrho = 10^{-23}$ gm cm^{-3} = $10^7 \langle \varrho \rangle$

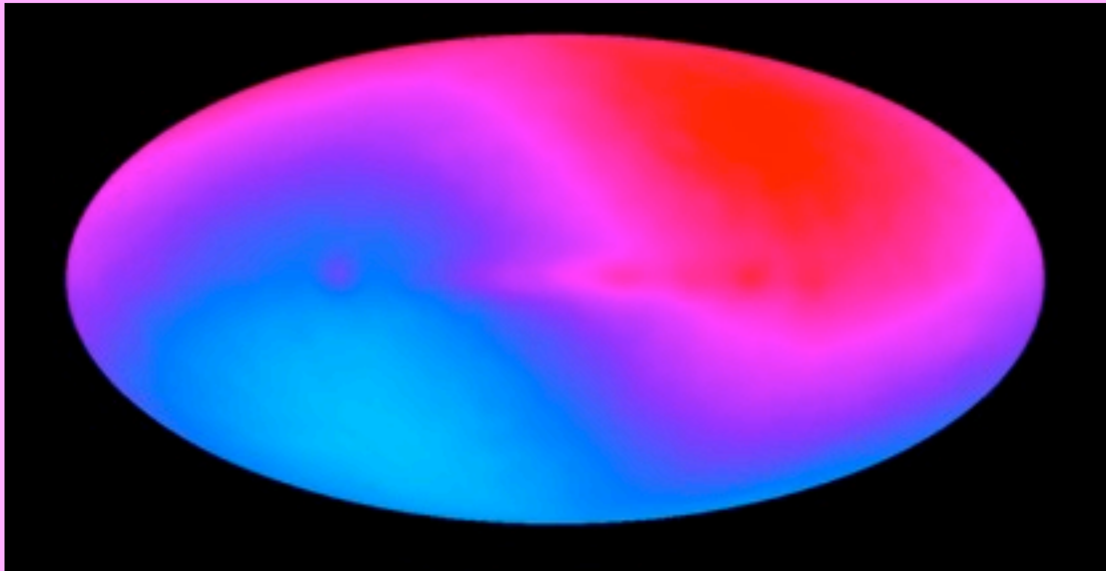
- Imprint on CMB

$$\frac{\delta\rho}{\rho} \sim \frac{\delta T}{T}$$

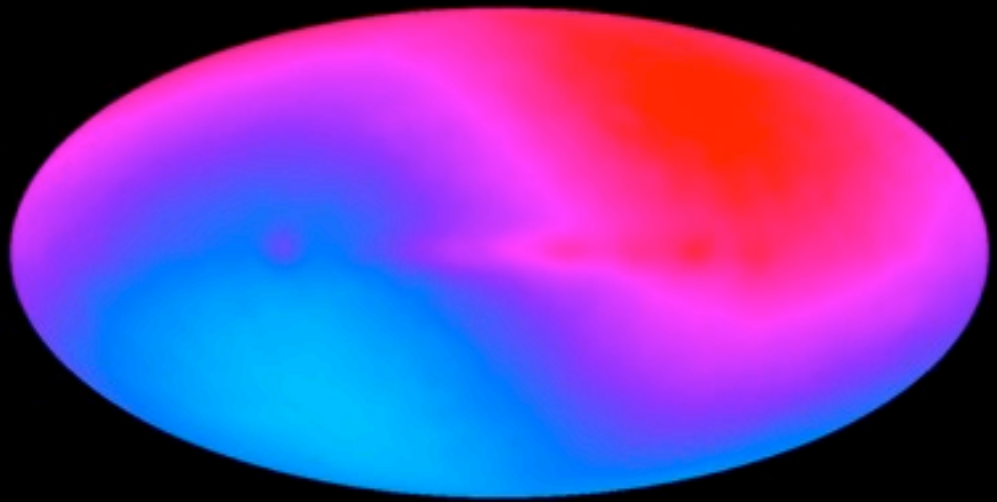
How big?

COBE anisotropy results

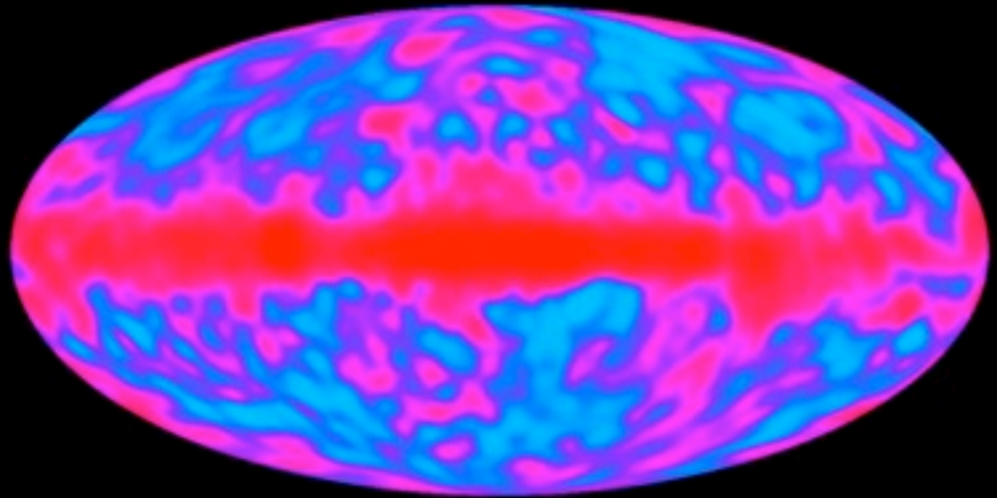
Dipole (monopole removed)



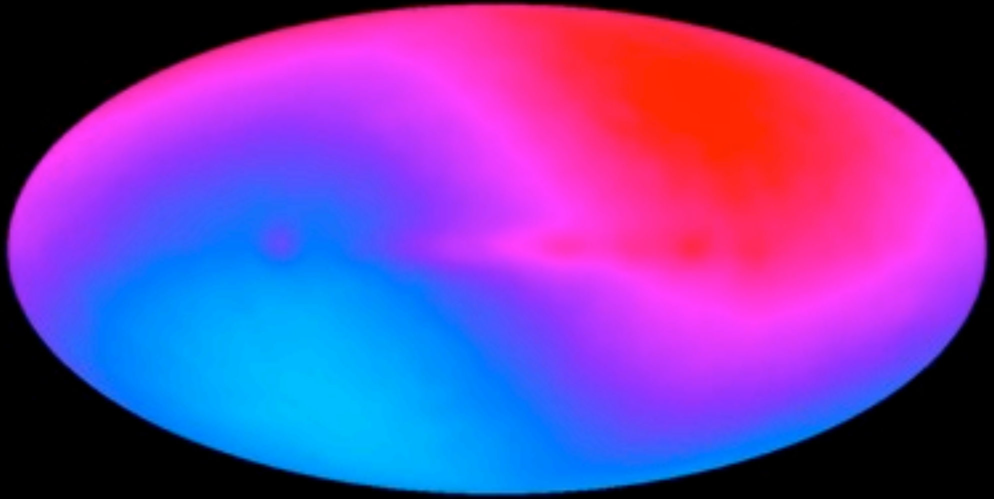
Dipole (monopole removed)



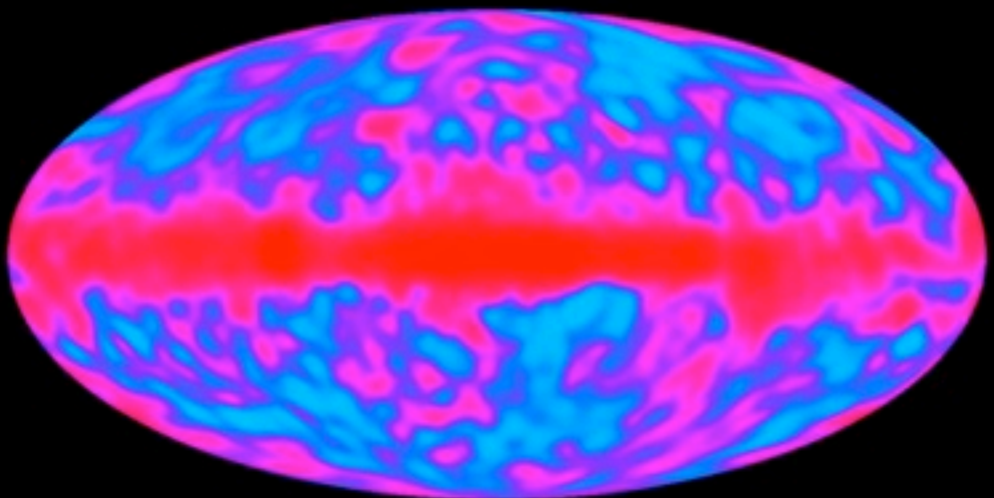
Dipole removed



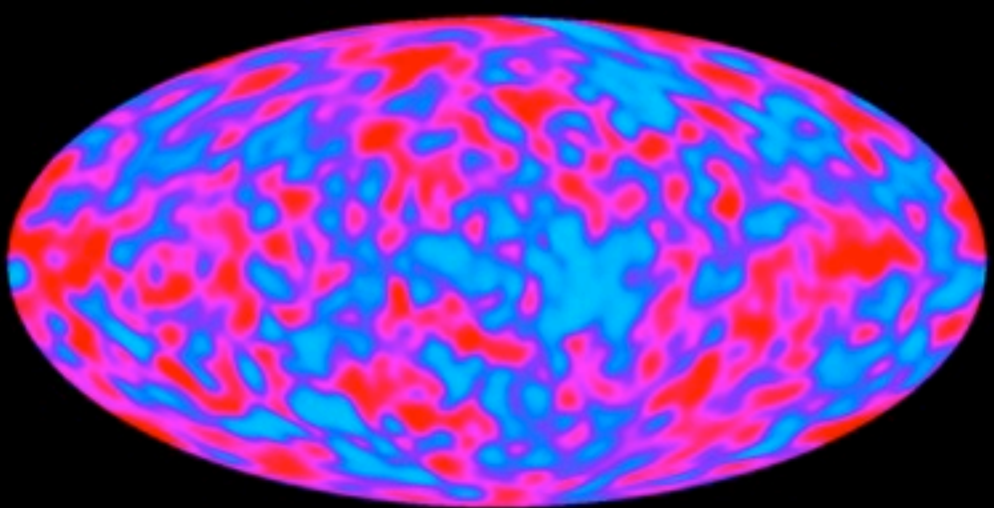
Dipole (monopole removed)

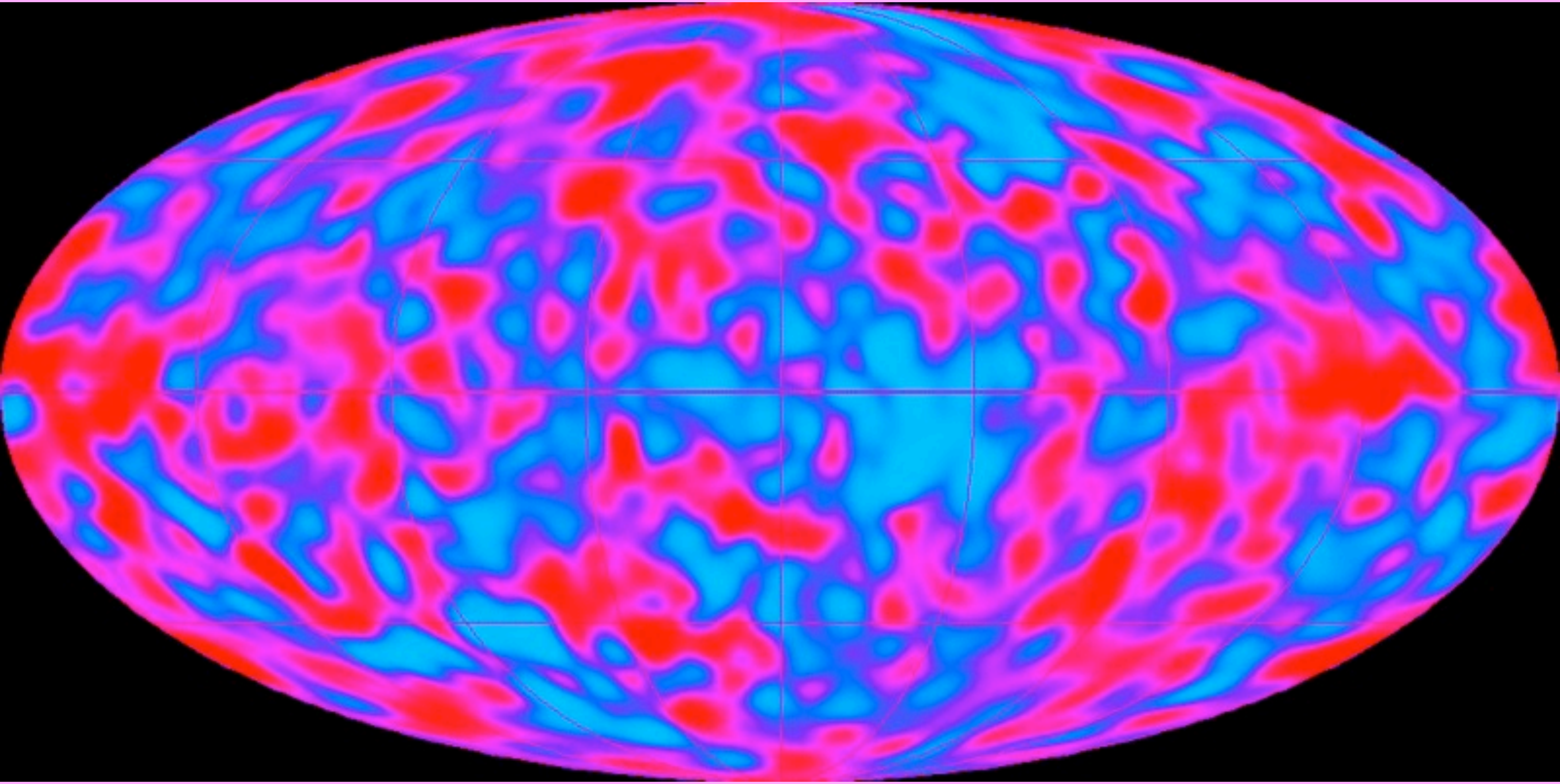


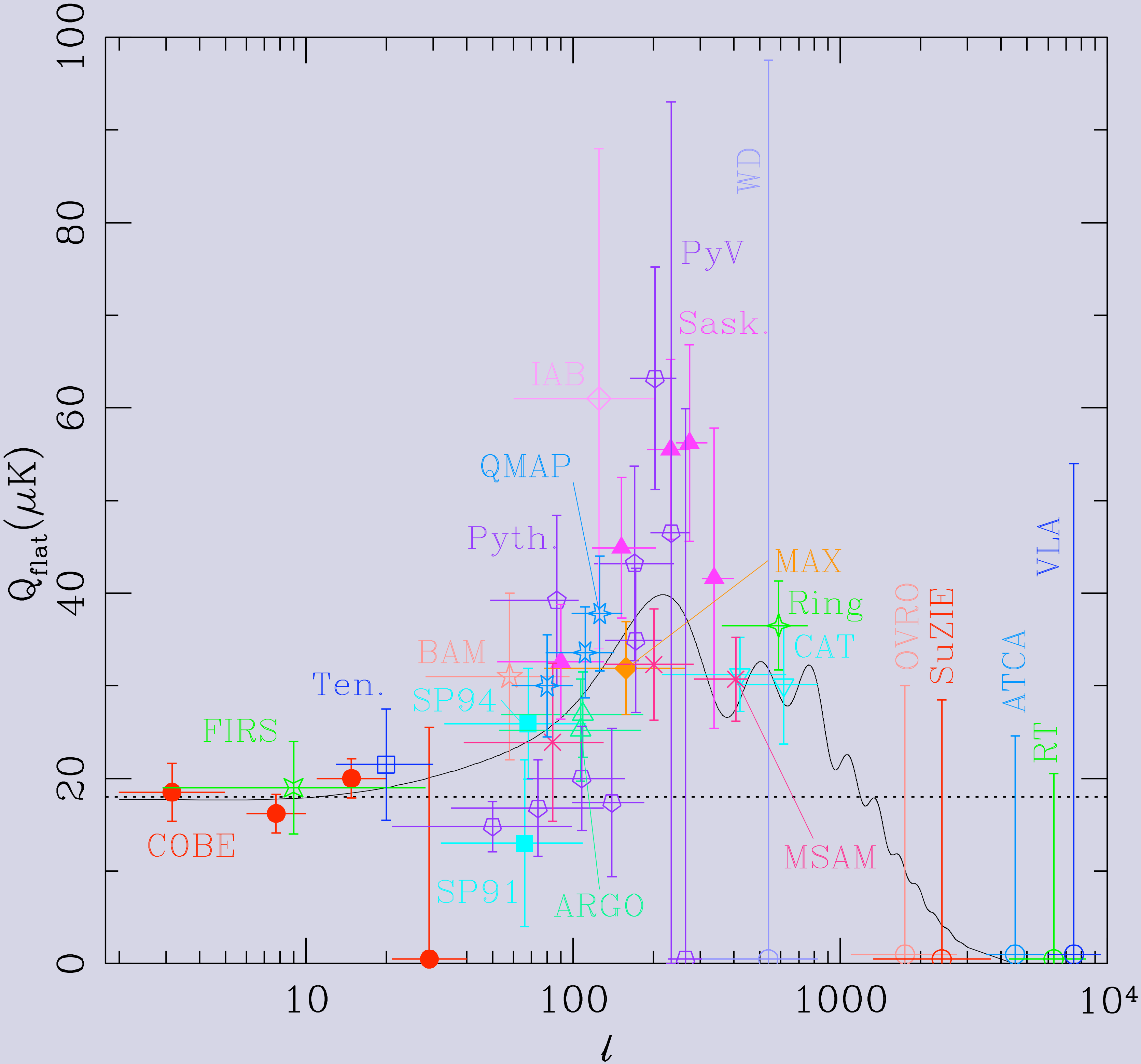
Dipole removed



Galaxy removed

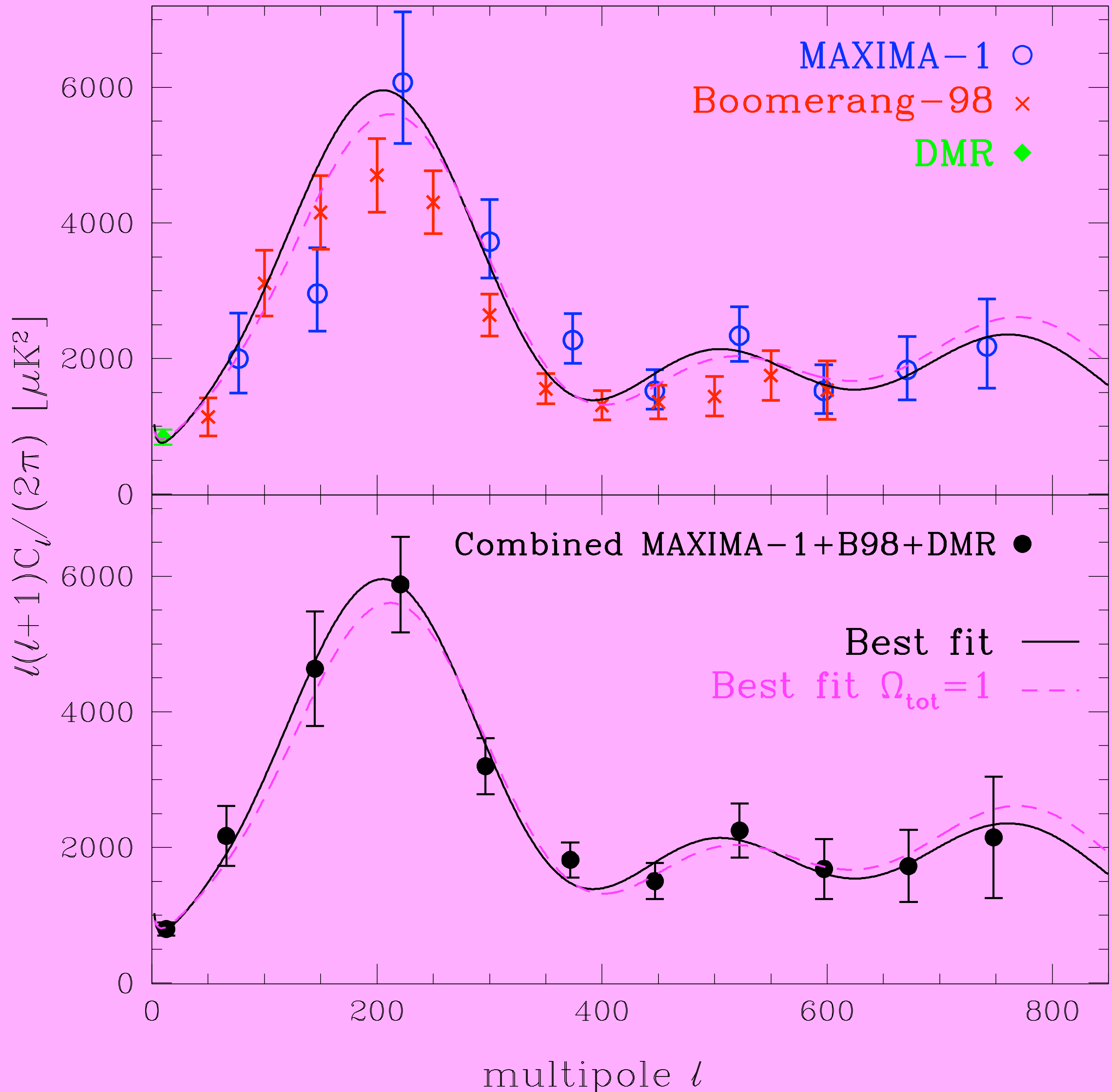






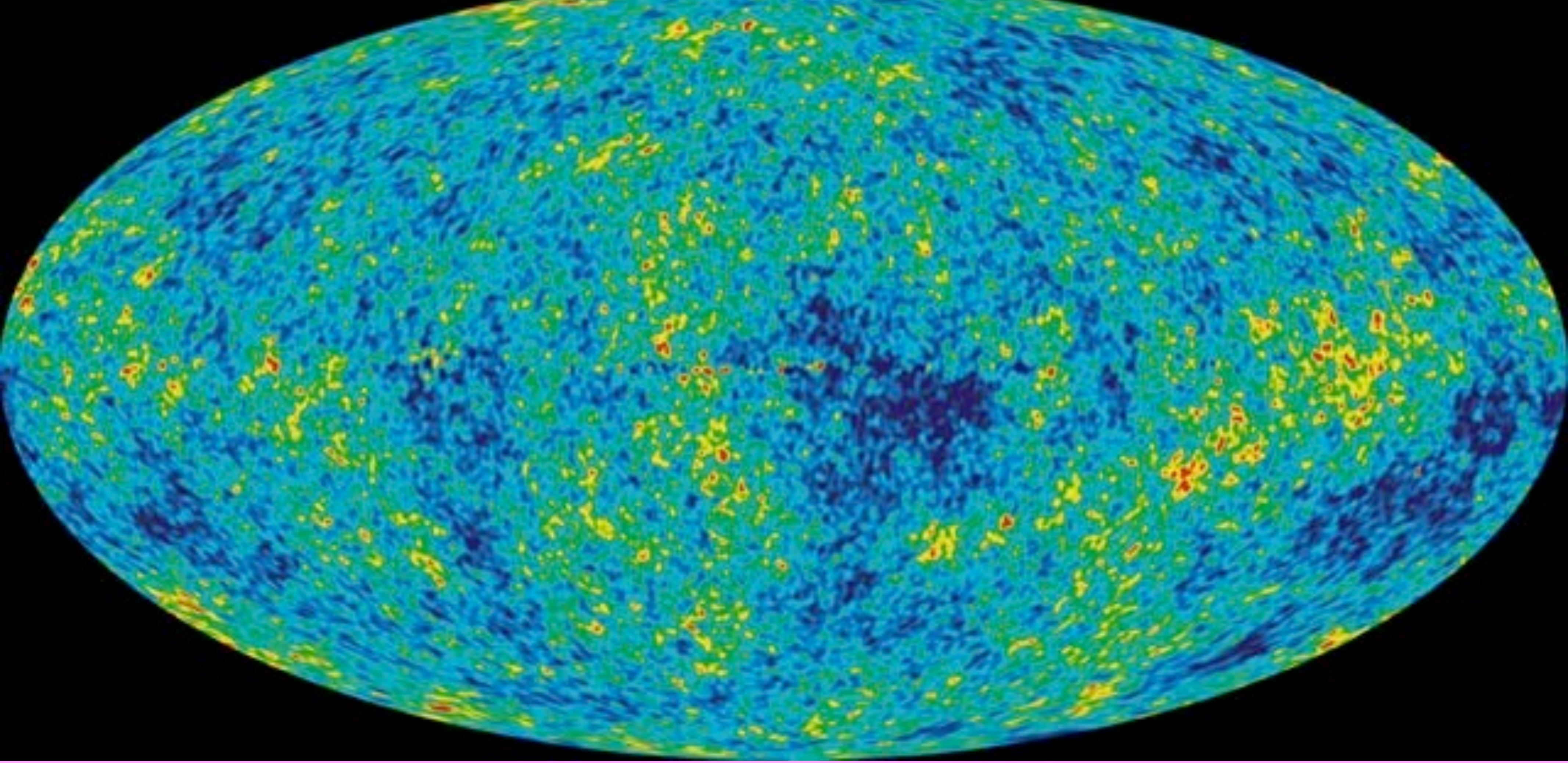
100
80
60
40
20
0
$Q_{\rm flat}(\mu K)$
10
100
1000
10^4
ℓ
COBE
FIRS
Ten.
SP94
SP91
BAM
ARGO
Pyth.
QMAP
IAB
PyV
Sask.
WD
MAX
Ring
CAT
MSAM
OVRO
SuZIE
ATCA
RT
VLA


NATIONAL
FACILITY



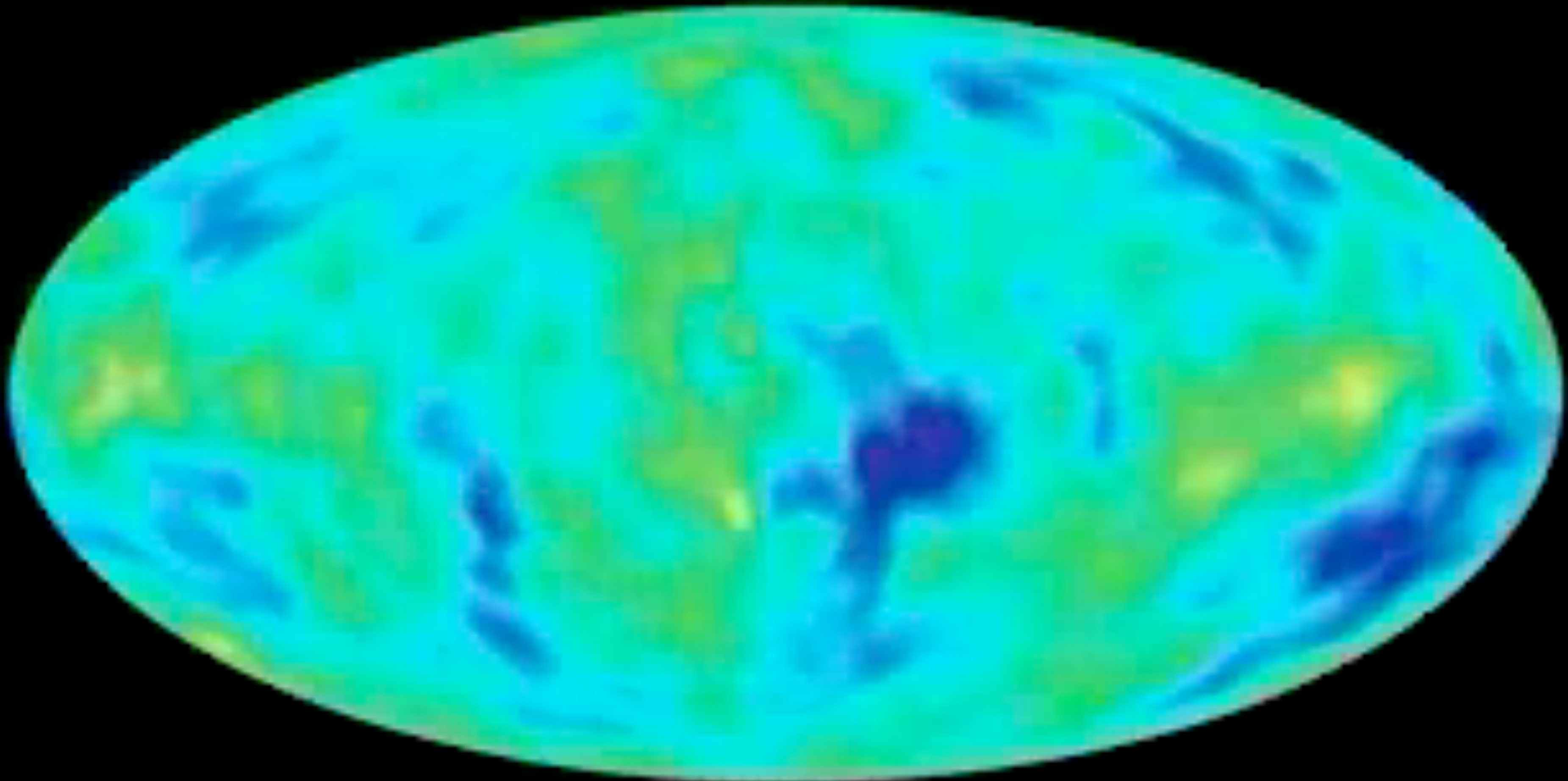
MAXIMA−1
Boomerang−98
DMR
Combined MAXIMA−1+B98+DMR
Best fit
Best fit $\Omega_{tot}=1$
$\ell(\ell+1)C_\ell/(2\pi)$ [μK^2]
multipole ℓ
0
2000
4000
6000
0
200
400
600
800


MAP990389



WMAP view of the microwave background anisotropy

WMAP Improvement in Resolution



The Power spectrum

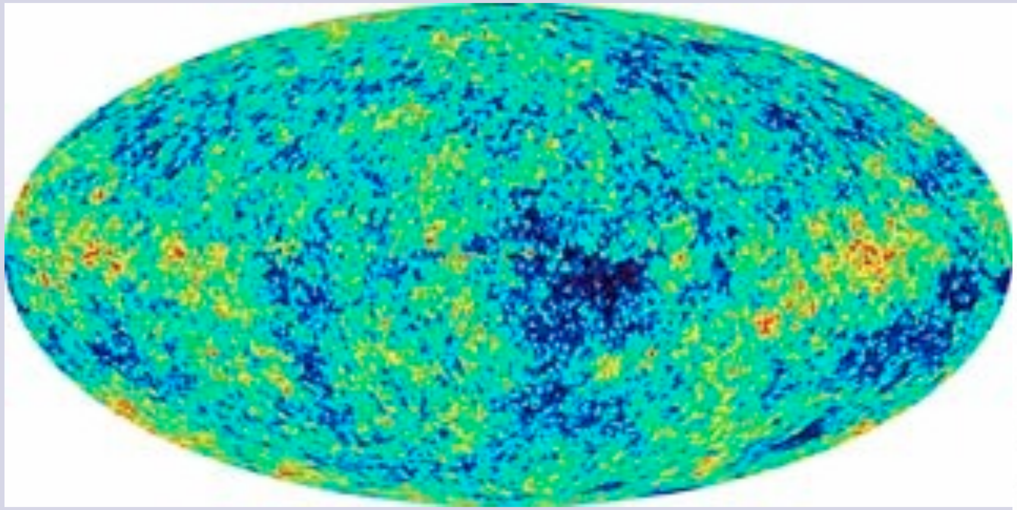
Expand Temperature map in spherical harmonics

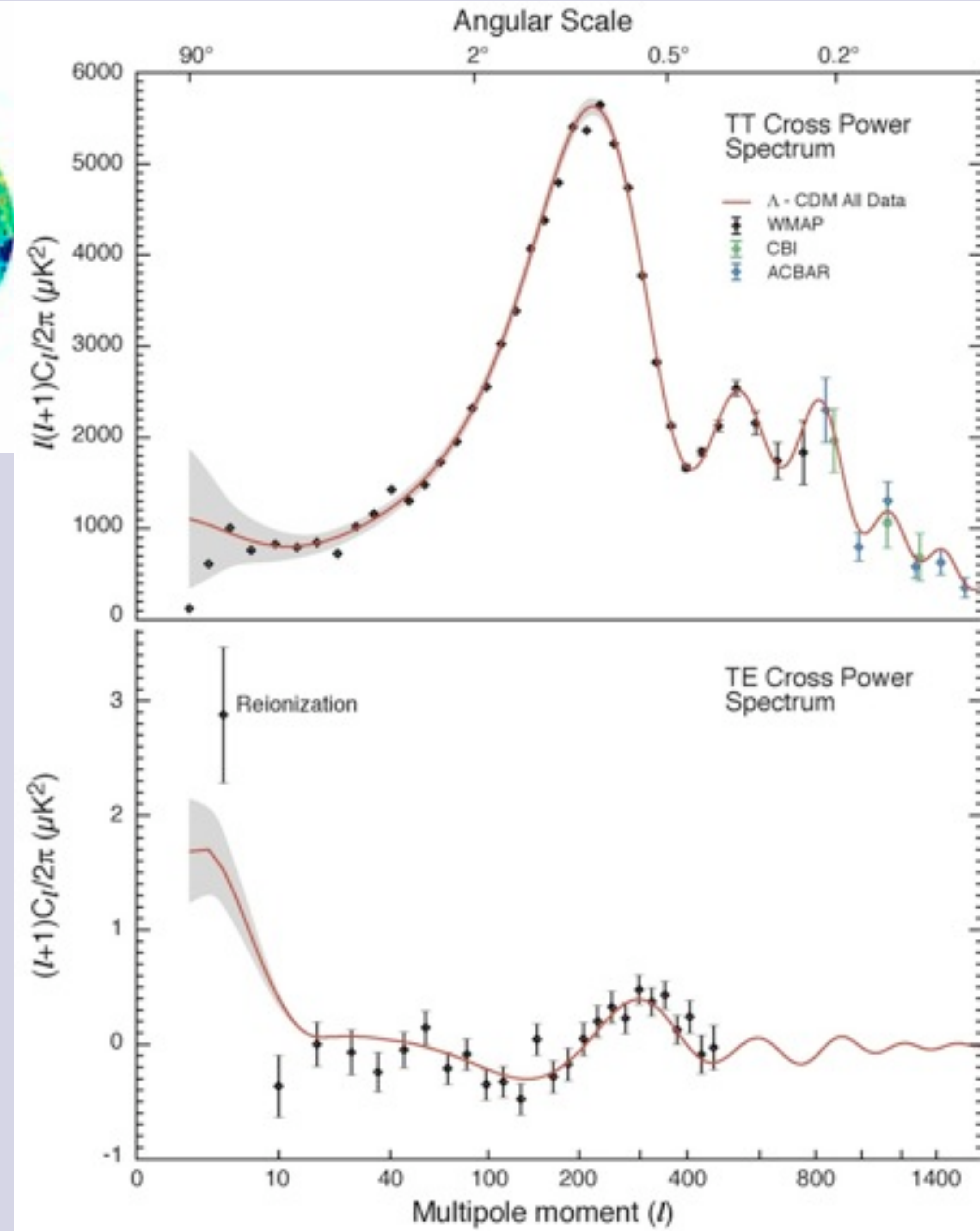
$$T\left(\theta,\phi\right)=\sum_{\ell m}a_{\ell m}Y_{\ell m}\left(\theta,\phi\right).$$

The power at each ℓ is $(2\ell+1)C_\ell/(4\pi)$,

where, $\quad C_\ell \equiv \left\langle |a_{\ell m}|^2 \right\rangle$

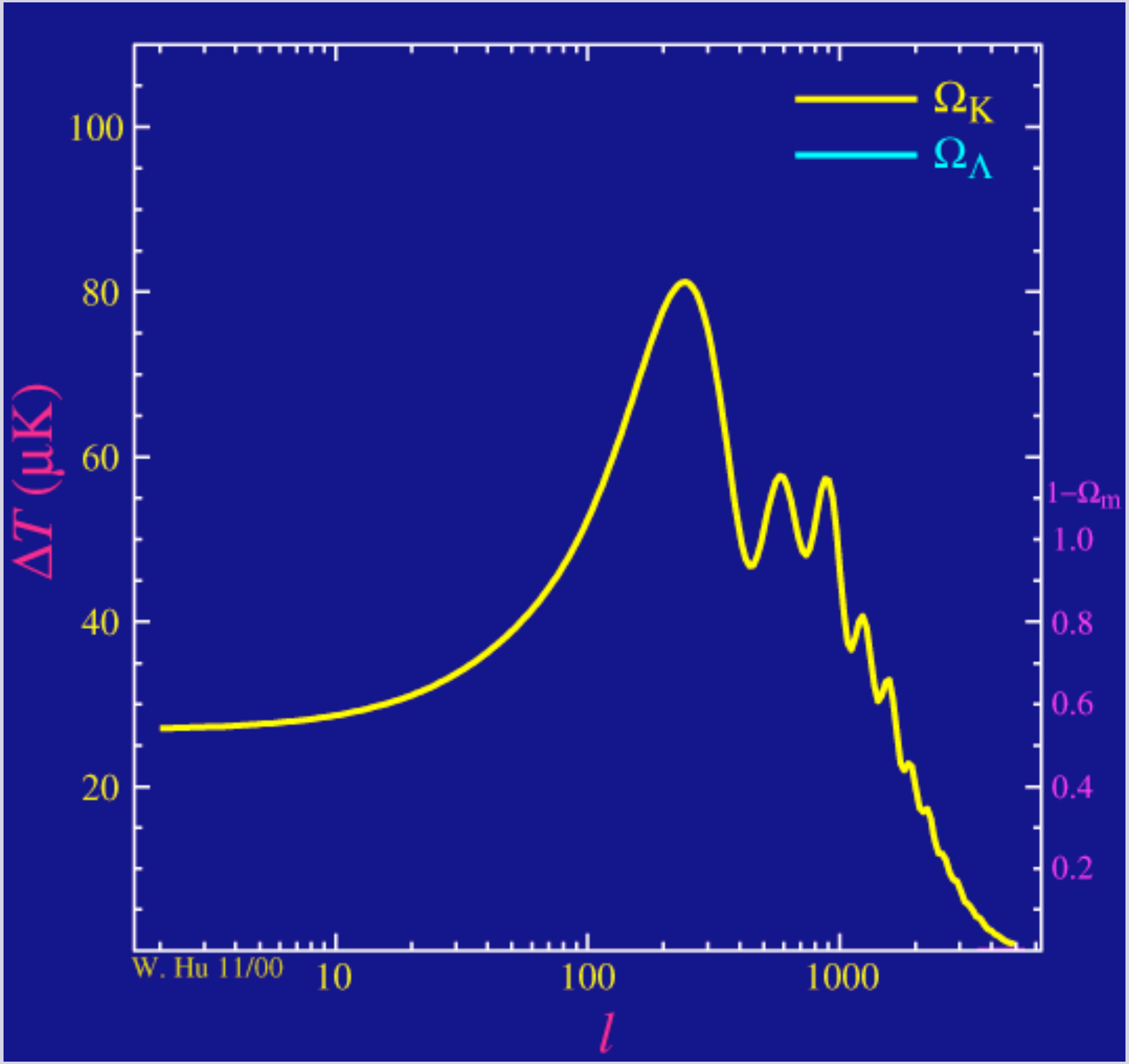
WMAP





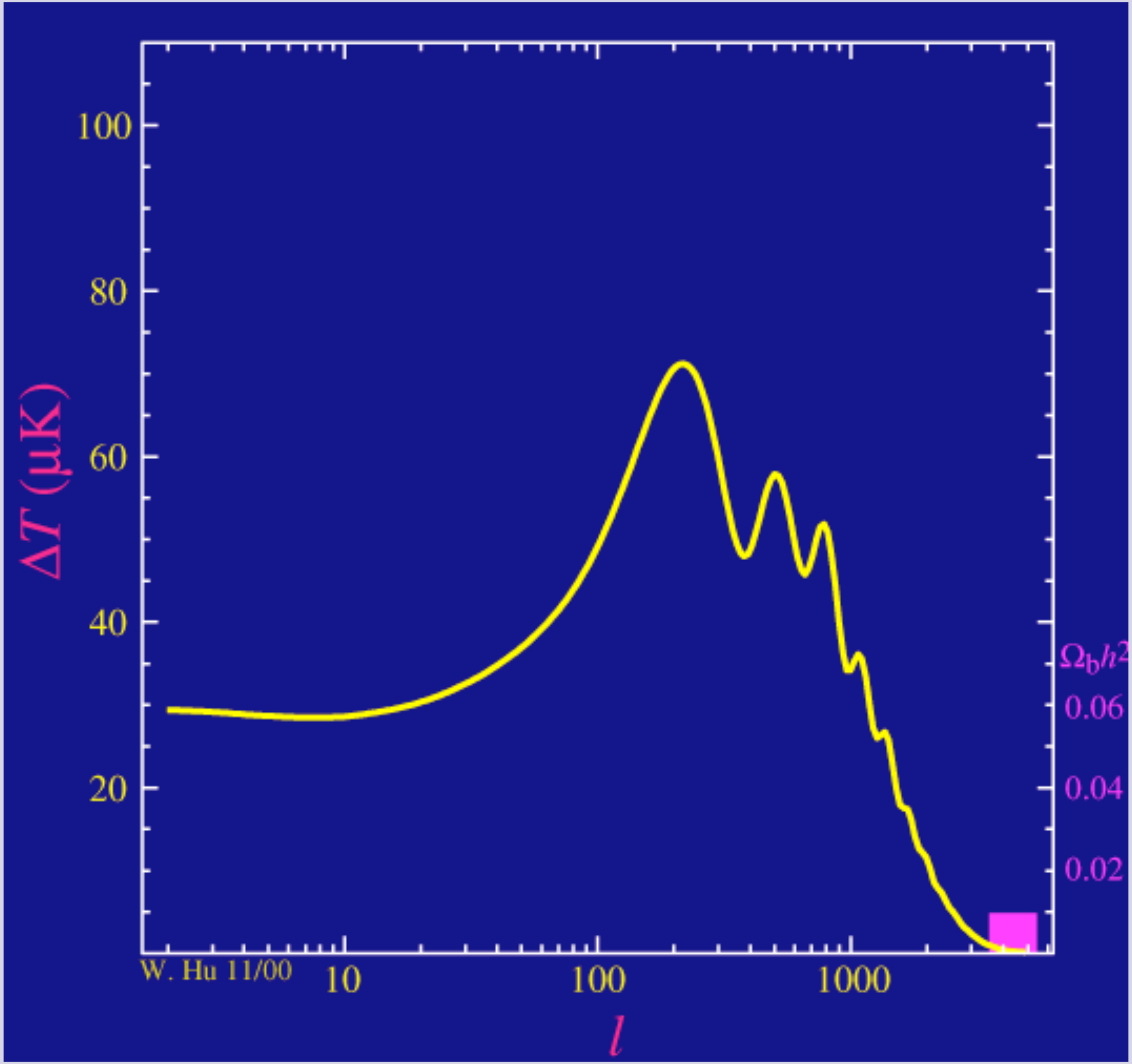
Position of 1st peak

$\Rightarrow \quad \Omega = 1$

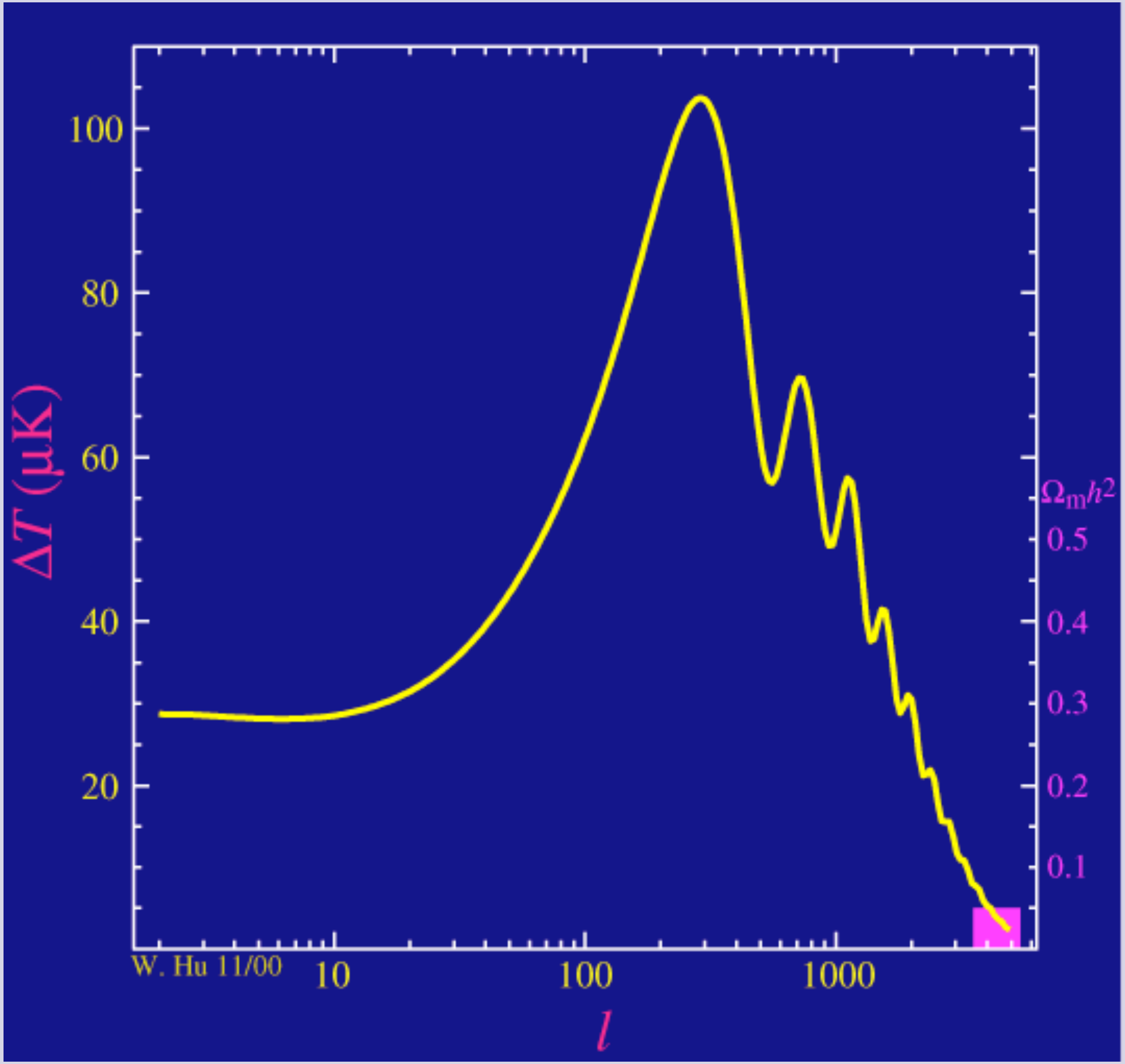


Ω_K
Ω_Λ
100
80
60
40
20
ΔT (μK)
$1-\Omega_m$
1.0
0.8
0.6
0.4
0.2
W. Hu 11/00
10
100
1000
l

Wayne Hu


100
80
60
40
20
ΔT (μK)
$\Omega_b h^2$
0.06
0.04
0.02
W. Hu 11/00
10
100
1000
l

Wayne Hu



100
80
60
40
20
ΔT (μK)
W. Hu 11/00
10
100
1000
l
$\Omega_m h^2$
0.5
0.4
0.3
0.2
0.1

Wayne Hu

Cosmological Parameters:

Hubble parameter	h
Cosmological constant	Ω_Λ
Dark matter density	Ω_{dm}
Baryon density	Ω_{B}
Radiation density	Ω_{rad}
Neutrino density	Ω_ν
Density perturbation amplitude	$\mathcal{P}_{\mathcal{R}}(k_*)$
Density perturbation spectral index	n
Tensor to scalar ratio	r
Ionization optical depth	τ

$$\Omega = 1.011 \pm 0.012$$

	WMAP alone	WMAP + 2dF	WMAP + all
$\Omega_{\mathrm{m}} h^2$	0.128 ± 0.008	0.126 ± 0.005	0.132 ± 0.004
$\Omega_{\mathrm{b}} h^2$	0.0223 ± 0.0007	0.0222 ± 0.0007	0.0219 ± 0.0007
h	0.73 ± 0.03	0.73 ± 0.02	$0.704^{+0.015}_{-0.016}$
n	0.958 ± 0.016	0.948 ± 0.015	0.947 ± 0.015
τ	0.089 ± 0.030	0.083 ± 0.028	$0.073^{+0.027}_{-0.028}$
σ_8	0.76 ± 0.05	0.74 ± 0.04	0.78 ± 0.03

Lahav + Liddle